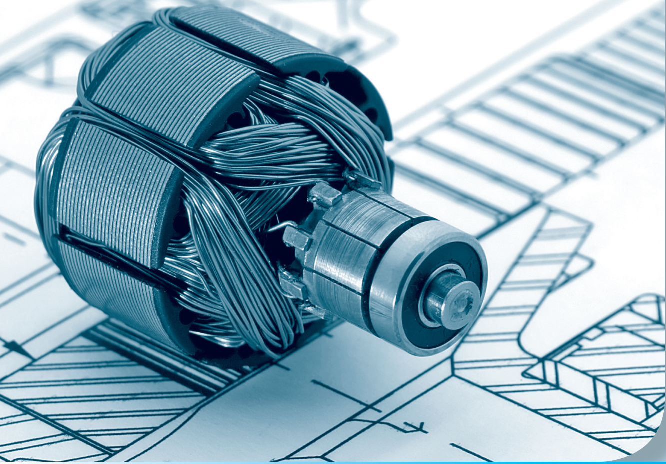


5

Synchronous Machines

What You Will Learn in This Chapter

A. Theoretical Aspects

1. Principle of operation of synchronous machine
2. Equivalent circuit
3. Phasor diagram
4. Power and torque produced
5. Effects of field current on motor characteristics
6. Three-phase cylindrical rotor generator
7. Generator characteristics
8. Measurement of their parameters
9. Three-phase salient-rotor machines
10. Phasor diagrams, torque developed

B. Outstanding Practical Highlights

1. Vee curves
2. How, by adjusting the field current, one can control a distribution system's power factor and reactive power
3. How to draw a step-by-step phasor diagram
4. Generator's voltage regulation
5. Manufacturer's motor-generator parameters
6. Operating speed and torque developed
7. Why the inductive loads in a synchronous generator weaken its field

C. Additional Student Aids on the Web

1. Three-phase faults and selection of breakers
2. Principle of constant-flux linkage
3. Stator currents
4. Field current

5.0 Three-Phase Synchronous Machines

Three-phase synchronous motors are used to drive loads that operate at constant speed. Such loads include, for example, the grinding mills of the mining industry and the rolling mills of the paper and textile industries.

Synchronous motors operating at 1800 r/min with ratings larger than 1000 kW have efficiencies that are as much as 4% higher than those of equivalent three-phase squirrel-cage induction motors.

Synchronous generators produce a balanced three-phase voltage supply at their output terminals by transforming the mechanical energy transmitted to their rotors into electrical energy. The source of the mechanical power commonly known as the prime mover may be wind, hydropower, or steam.

Synchronous generators are used mainly by utility companies to generate their standard, three-phase, 60 Hz power. In fact, most ac power is generated through synchronous generators, the ratings of which sometimes exceed 1000 MVA. In consumers' plants, smaller machines (up to 1 MVA) provide standby emergency power to critical loads in the event the utility's power system fails. Remote locations use synchronous generators with ratings from 150 watts to several kilowatts.

Increasingly, small synchronous generators—down to 20 kW—are being installed where a source of power can produce electricity economically. Such power may be supplied, for example, by small hydroelectric plants, or they may derive from the excess steam from chemical processes, the heat produced by factories burning dried cotton, and the like. Most automobiles are also equipped with three-phase generators.

Synchronous machines are classified as either *cylindrical rotor* or *salient rotor*. (Machines of both types will be discussed in the sections that follow. The three-phase short circuits of the machines are discussed in the Web section.)

The control schematics and the modern electronic control of synchronous machines are explained in Chapter 7.

5.1 Three-Phase Cylindrical Rotor Machines: Motors

Three-phase synchronous motors are externally controlled, variable-power-factor machines. They run at synchronous speeds, and, as shown in Fig. 5-1(a), they require two sources of excitation:

1. A balanced three-phase ac source for the stator windings.
2. A dc source for the rotor windings.*

The structure and winding arrangement of the stator of a synchronous motor is identical to that of a three-phase induction motor. The rotor, however, is of the wound type and may be of either cylindrical or salient construction.

The dc current for the rotor windings may be obtained either from an external dc source through the slip rings of the rotor (classical machines) or from the rectified ac voltage induced in a special set of rotor windings (modern or brushless machines). In the second case, in addition to the dc *field winding,* the rotor carries a three-phase *ac winding* and a set of *solid-state devices* (thyristors and diodes). These solid-state devices are located between the ac and dc windings. When the rotor rotates, the magnetomotive force of an external dc supply induces a voltage in the ac winding.

Solid-state devices rectify the induced voltage and control the timing of the application of the rectified voltage to the dc windings. (A detailed description of the dc voltage source for the field windings of the motor is given in Section 7.3.2.)

Brushless synchronous motors also include those synchronous machines that obtain their field excitation from an auxiliary generator, commonly known as an exciter. The exciter often is driven by the same prime mover as the synchronous machine.

*Permanent-magnet rotors may be used in small machines.

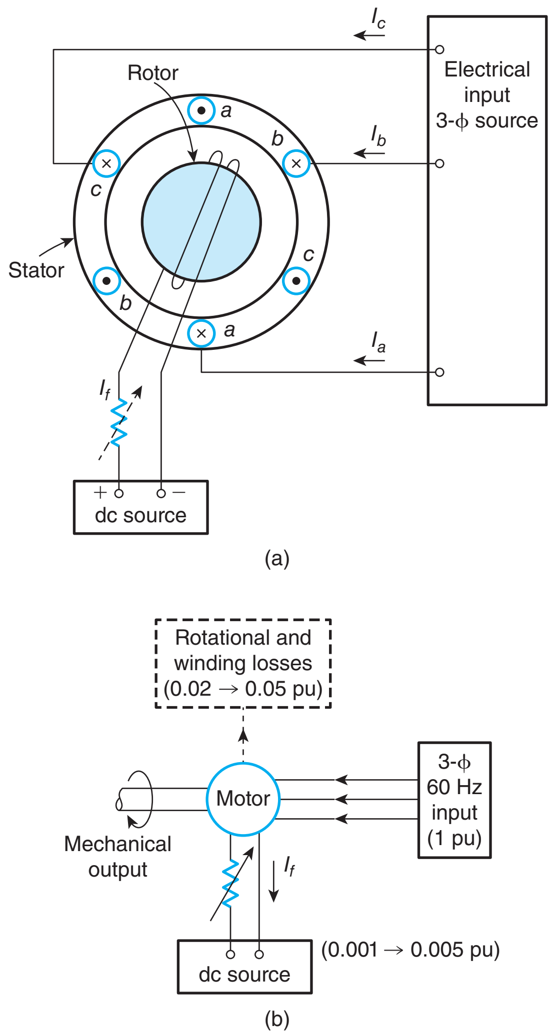


FIG. 5-1 Synchronous motor: **(a)** elementary representation of stator and rotor windings–cylindrical rotor, **(b)** direction of power flow and typical losses for a 100 kW machine.

The motor's mechanical output power comes from transformation of the input ac power. Although the dc supply is essential for operation of the motor, it transfers no power to the output. Figure 5-1(b) shows typical values of the input power and losses for a 100 kW motor.

The electrical input to the rotor windings of large synchronous machines is less than 1% of the total power. In efficiency calculations, therefore, it may be neglected. The field current essentially controls the reactive volt-ampere requirements of the stator circuits and has a pronounced effect on the motor's output torque.

In synchronous machines, the magnetic flux of the armature may aid or oppose the magnetic flux of the field current. This interaction, termed *armature reaction*, depending on its effects, is also called the magnetizing or demagnetizing effect of the armature current. Determining the effects of the armature current is essential in analyzing and understanding synchronous machines.

The main advantage of synchronous motors is that their power factor (cos θ), input line current (I_L), and input reactive power (Q) can be easily controlled by properly adjusting the field current. In other words, synchronous motors, as seen from the network terminals, behave as either variable-shunt capacitors or variable-shunt inductors. This topic is discussed in detail in Section 5.1.9.

Synchronous motors provide variable torque for loads that require a constant speed. For ordinary applications, however, they are neither economical nor practical because they operate at only one speed and require an ac source for stator excitation and a dc current for their rotor windings. Besides this drawback, they have another crucial disadvantage: Sudden or cyclic variations in the mechanical load torque may drive the motor out of synchronism. This problem may cause disturbances and lead both the motor and the supplying electrical system to shut down.

A major engineering problem related to the steady-state operation of synchronous motors involves calculating the *field current* that corresponds to a specific armature current and power factor.

The following sections discuss the cylindrical-rotor motor's principle of operation, equivalent circuit, governing equations, phasor diagrams, and external characteristics, commonly known as Vee curves. The photographs shown in Fig. 5-2 to Fig. 5-5 show some physical characteristics of synchronous motors.

FIG. 5-2 A four-pole, 15 MW, 60 Hz synchronous motor.
Courtesy of General Electric

FIG. 5-3 Stator of a 2.6 MW synchronous motor. *Courtesy of General Electric*

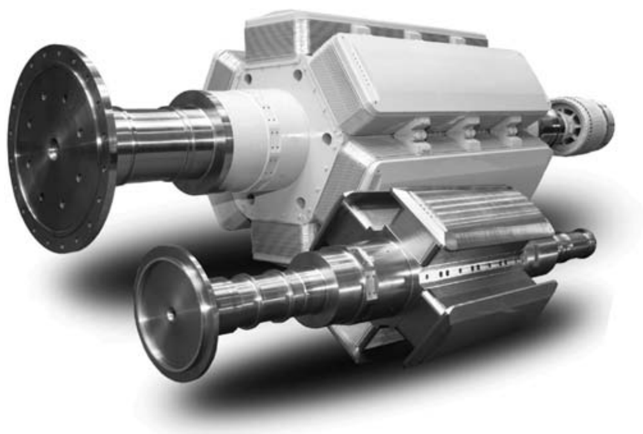


FIG. 5-4 Rotor of a 2.6 MW synchronous motor. *Courtesy of General Electric*

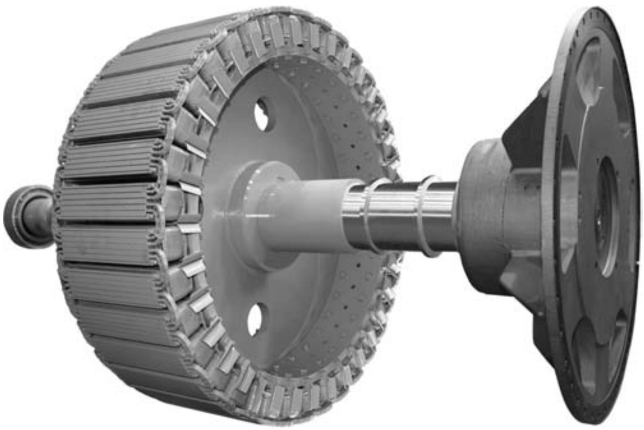


FIG. 5-5 Salient-rotor synchronous motor: Continuous-end rings. *Courtesy of General Electric*

5.1.1 Rotor

An elementary representation of a synchronous motor with a cylindrical rotor is shown in Fig. 5-1(a). In general, motors with rated speeds of 1800 or 3600 r/min have cylindrical rotors, whereas synchronous motors with lower rated speeds are designed, for economical reasons, with salient rotors.

Cylindrical-rotor machines are relatively simple to analyze; they simply cease to operate when their field current is removed. Salient-pole machines, however, are more difficult to analyze. Without any excitation, they may develop a torque that may be up to 40% of the rated torque.

5.1.2 Rotating Fields

Synchronous machines have two synchronously rotating magnetic fields:

1. The dc field of the rotating rotor.
2. The stator or armature field.

The stator field is identical to that produced by the armature of three-phase induction motors. For convenience, its description is repeated here.

Refer to Fig. 5-6(a). The magnitude and speed of rotation of the stator field are given by

$$\mathcal{F}_s = \frac{3}{2}\mathcal{F}_1 \cos(\omega t - \beta) \text{ ampere-turns} \tag{5.1}$$

$$\eta_s = 120\frac{f}{p} \text{ r/min} \tag{5.2}$$

where:

$\mathcal{F}_s$ = is the rotating stator mmf produced by the three-phase armature currents

$\mathcal{F}_1$ = is the maximum value of the mmf produced by one phase only. For balanced armature currents and an equal number of turns per winding, we have

$$\mathcal{F}_1 = N_a I_{am} = N_b I_{bm} = N_c I_{cm} \text{ ampere-turns} \tag{5.3}$$

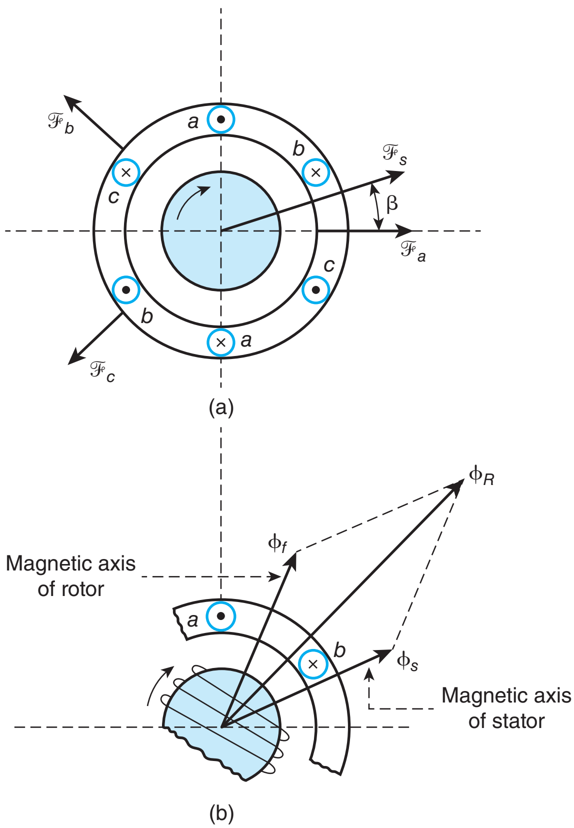


FIG. 5-6 Magnetic fields in a synchronous motor: **(a)** stator mmf's, **(b)** stator, rotor, and resultant magnetic flux.

β = an arbitrary angle measured with respect to the horizontal line, as shown in Fig. 5-6(a).

ω = the angular speed of the stator currents

f = the frequency of the stator currents

η_s = the synchronous speed of the rotating field

p = the number of poles of the motor.

Equations (5.1) and (5.2) are derived in Section 3.1.3. The magnetic field of the armature is equivalent to the field of a synchronously rotating permanent magnet. Under normal operating conditions, this field does not induce any voltage in the rotor winding because the rotor winding is rotating at the same speed.

As shown in Fig. 5-6(b), the field of the rotor is along the magnetic axis of the rotor. Since the rotor rotates at synchronous speeds, under normal operating conditions, its field also travels at the same speed.

Under normal operating conditions, then, the stator and rotor fields travel at the same angular speed, which is known as the synchronous speed and is given by Eq. (5.2).

5.1.3 Principle of Operation

The motor's operation, or torque-producing capability, results from the natural tendency of two magnetic fields to align their magnetic axes. The fields under consideration are:

1. The stator field (ϕ_s).
2. The rotor field (ϕ_f).

The flux that accompanies the synchronously rotating stator field (ϕ_s) completes its magnetic circuit by passing through the stator, air gaps, and rotor. The flux of the rotor field (ϕ_f) also passes through the rotor, the air gaps, and the structure of the stator. The vector sum of the armature and field flux, as shown in Fig. 5-6(b), gives the so-called resultant (ϕ_R) air-gap flux. Mathematically,

$$\vec{\phi}_R = \vec{\phi}_s + \vec{\phi}_f \tag{5.4}$$

The magnetic axis of the field flux tries to align itself with the magnetic axis of the stator flux. This process produces motor action. The mathematical expression for the torque developed was derived in Chapter 3 on three-phase induction machines (see Section 3.1.2). This electromagnetic torque is in the direction of rotation of the stator field. When an additional load is applied to the shaft, the rotor or the magnetic axis of the field flux slips in space behind the magnetic axis of the resultant air gap flux. The result is a larger space angle; therefore, a larger torque is developed.

5.1.4 Starting

Starting a three-phase synchronous motor is similar to starting a three-phase induction motor. Balanced voltages are applied to the stator windings, while the rotor winding is short-circuited or connected to a resistance. Synchronous motors are commonly equipped with damper windings, which are similar to the rotor windings of squirrel-cage induction motors. During any nonsynchronous-speed operation, damper windings contribute to the motor's output torque. The stator voltages produce a rotating magnetic field, which induces a voltage in the rotor winding. This voltage will circulate rotor current, which in turn will produce a magnetic field.

The interaction of these two fields produces motor action, which will bring the rotor up to a speed that is very close to synchronous. At this instant, the rotor windings are switched to the normal dc supply, and the rotor pulls in synchronism with the armature rotating field. In practice, this switching often takes place at that particular instant when the flux of the field momentarily reduces the voltage already impressed in the stator. This results in high inrush currents in the armature windings, which are accompanied by a characteristic "thump."

The resistance (R_2) of the rotor circuit limits the rotor current and thus controls the starting torque of the motor. Generally speaking, the higher the resistance of the field windings, the higher the starting torque. Mathematically,

$$T_{st} \propto R_2 \tag{5.5}$$

The torque that the motor develops, at the instant the dc field is "switched in," is usually referred to as the pull-in torque. The pull-in torque must be larger than the load requirements at that speed; otherwise, the motor will not be able to synchronize.

At starting, synchronous motors have very low impedance. As a result, their starting current is five to six times greater than their rated current. Depending on the Thévenin's impedance of the supply network, this high current may cause a dip in the voltage delivered to the plant where the motor is operating. This voltage reduction will reduce the starting-torque capability of the motor ($T \propto V^2$) and may adversely affect other electrical loads, such as motors, lights, and magnetic contactors.

The accelerating time, the magnitude of the inrush current, and the torque are identical to those of three-phase induction machines.

Where circumstances do not permit high inrush currents, an autotransformer may be used to reduce the voltage applied to the motor. A more popular method is to design a motor with low starting torque and inrush current. Such a motor is coupled to the load through an air clutch. The motor starts unloaded, and after it is synchronized, the pressurized air—which is conducted through a properly sized opening along the inner section of the motor's shaft—engages or couples the air clutch to the load. Use of an air clutch not only permits the design of motors with lower starting torque and currents, but also develops pneumatic power that is sufficient to provide the high starting torque that some loads require.

The appendix to this chapter gives manufacturers' data for high- and low-starting-torque (soft or air-clutch starting) synchronous motors.

5.1.5 Equivalent Circuits

This section develops and analyzes the approximate equivalent circuit(s) of a synchronous motor. The flow of energy from the ac and dc sources and the electromagnetic coupling between their corresponding fields are shown in Fig. 5-7(a). This electromagnetic coupling leads to the development of torque. From Ampere's law, the torque developed depends on the magnitude and relative position of the two interacting fields.

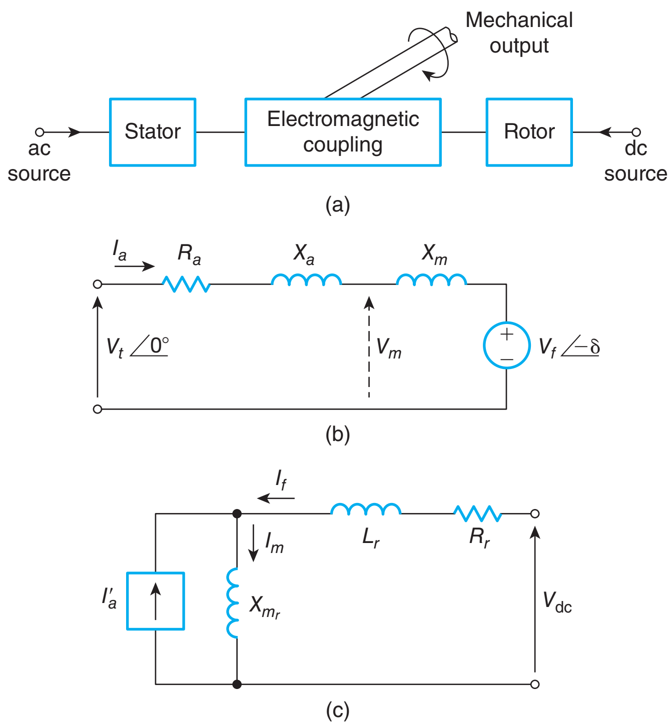


FIG. 5-7 Synchronous motor: **(a)** elementary presentation of energy flow, **(b)** approximate per-phase equivalent circuit as seen from the stator, **(c)** approximate equivalent circuit as seen from the rotor.

The approximate per-phase equivalent circuit of a motor, as seen from the stator (Fig. 5-7(b)), is derived by considering the effects of the two rotating fields on the stator. The resultant field magnetizes the stator (a process represented by X_m). The rotor field induces a voltage (V_f) in the stator windings whose phase angle is at $-\delta°$ to the stator voltage supply (V_t) because the synchronously rotating rotor field lags the stator field by the same angle. The leakage impedance of the stator windings is represented by $R_a + jX_a$.

The approximate equivalent circuit of a motor as seen from the rotor (Fig. 5-7(c)) is derived by considering the effects of the two rotating fields on the rotor. The resultant field magnetizes the rotor (a process represented by the reactance X_{mr}), and the effects of the stator current are represented by a current source of I'_a amperes. The actual resistance and inductance of the rotor windings are as shown in the figure.

The stator and rotor equivalent circuits supplement each other. They give an in-depth view of the machine's conceptual design and behavior, enabling the application engineer to predict the motor's operation.

Equivalent-Circuit Parameters

The physical significance and nomenclature of the various parameters shown in the equivalent circuits are as follows.

Stator Parameters

V_t = *terminal voltage*. This is the per-phase ac stator voltage. Its magnitude depends on the voltage rating of the motor and on the type of connection (delta or star) of the armature windings. In phasor diagrams, or in actual calculations, this voltage is taken as reference.

V_m = *magnetizing voltage*. This is the voltage induced in the stator windings by the effective flux of the motor. This flux depends on the resultant mmf (vector sum of the stator and rotor's mmf's) and on the degree of magnetic saturation.

V_f = *excitation voltage*. This is the voltage induced in the armature windings by the dc rotating field. In the absence of saturation, the excitation voltage is equal to the open-circuit voltage that is measured across the armature terminals when the machine is driven as a generator. This voltage is sometimes referred to as the machine's internal voltage or the open-circuit voltage. In motors, the excitation voltage (as explained in subsequent sections) always lags the supply or terminal voltage (V_t). Depending on the power factor, the excitation voltage may be larger or smaller than the terminal voltage. The excitation voltage is opposite in polarity to the terminal voltage, and depending on the motor's power factor, it may be smaller or larger than the supply voltage.

X_m = *magnetizing reactance* as seen from the stator. This parameter represents the magnetization of the stator that results from the effective field of the armature. It depends on the characteristics of the magnetic material and the degree of saturation.

X_a = *stator leakage reactance*.

R_a = *stator resistance*. This is the effective or ac resistance of the armature windings—that is, the stator dc resistance adjusted to include magnetic losses and the effects of higher operating temperatures.

X_s = *synchronous reactance*. This is the sum of the stator leakage reactance and the magnetizing reactance. That is,

$$X_s = X_a + X_m \tag{5.6}$$

Z_s = *synchronous impedance*.

$$Z_s = R_a + jX_s \tag{5.7}$$

I_a = *armature current*. When the motor is star-connected, this current is equal to the line current.

Power Angle (δ)

The power angle, designated by the Greek letter delta (δ), is the phase angle between the excitation voltage (V_f) and the magnetizing voltage (V_m). It is also called the torque or the load angle. This angle is equal to the phase angle between the resultant air-gap flux (ϕ_R) and the field flux (ϕ_f).

The phase angle between the terminal voltage (V_t) and the excitation voltage (V_f) is approximately equal to the torque angle because the leakage impedance is very small compared to the magnetizing reactance.

The torque angle for a synchronous motor is always negative (V_f lags V_t), regardless of operating power factor. The torque angle used for the steady-state analysis of the motors (and shown in the phasor diagrams) is expressed in electrical degrees. However, the torque angle used for analysis of the machine's electromechanical transient is expressed in mechanical degrees. The relationship between mechanical (δ_m) and electrical degrees (δ) is derived in Section 3.1.3 and is rewritten here for convenience:

$$\delta_m = \frac{2}{p}\delta \tag{5.8}$$

where p is the number of poles of the machine.

Typical ranges of the parameters of synchronous motors are given in Table 5-1.

TABLE 5-1 Typical synchronous machine parameters

Parameter	*Minimum**	*Maximum**
X_a	0.08	0.30
X_m	0.4	2.4
R_a	0.002	0.04

**All values are in per unit, based on the machine's rating.*

Rotor Parameters

V_{dc} is the dc voltage applied across the rotor windings. In all modern machines, this voltage is obtained by rectifying the voltage induced in another set of rotor windings from a source that is magnetically coupled to the rotor but not electrically connected to it.

X_{mr} is the machine's equivalent magnetizing reactance as seen from the rotor terminals.

R_r and L_r are the rotor resistance and self-inductance, respectively.

I_f is the dc current of the rotor winding.

I_m is the component of the field current that flows through the magnetizing reactance X_{mr}.

I'_a is the component of the stator current that represents the armature reaction. This is the contribution of the armature current to the magnetization of the machine; it is expressed in equivalent field amperes. Depending on the power factor at which the motor draws power, the armature reaction may aid or oppose the actual field current. (For an illustration, see the Phasor diagrams of Fig. 5-10.)

The ratio of the armature current (I_a) to its armature reaction component gives the effective turns ratio (N_e) between the stator and the rotor windings. Mathematically,

$$N_e = \frac{I_a}{I'_a} \tag{5.9}$$

The measurement of a machine's effective turns ratio and armature reaction is discussed in Section 5.2.3.

5.1.6 Field Current

From the equivalent circuit of Fig. 5-7(c), the motor's magnetizing current (I_m) is given by

$$I_m = I_f + I'_a \tag{5.10}$$

where I_f is the actual field current and I'_a is the contribution of the armature current to the magnetization process. Rewriting the last relationship, we have

$$I_f = I_m - I'_a \tag{5.11}$$

I'_a is in phase with the armature current and is related to it by the effective turns ratio, as previously mentioned. The field current and its magnetizing component lag their corresponding voltages by 90°, as shown in Fig. 5-8.

The field current and its magnetizing component can be obtained graphically by projecting their corresponding voltages on the machine's open-circuit characteristic (OCC). See Fig. 5-9.

The actual values of V_f and V_m at any particular motor operating condition can be found by applying KVL to the equivalent circuit of Fig. 5-7(b). The measurement of the open-circuit characteristic, and further applications, are discussed in detail in Section 5.2.3.

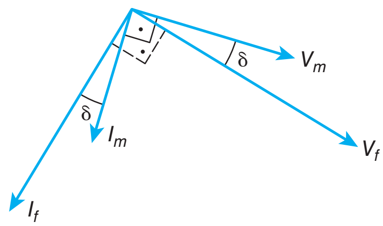


FIG. 5-8 Excitation and magnetization voltage phasors and their corresponding currents.

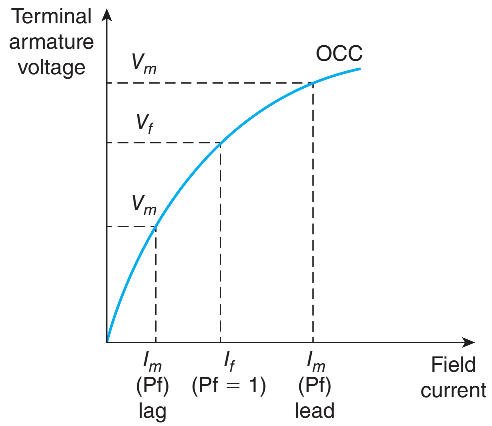


FIG. 5-9 The effects of the power factor on the magnetization voltage and on the magnetizing component of the armature current.

Equation (5.10) may be written in terms of the interacting mmf's. That is, the effective magnetizing mmf is equal to the mmf of the field plus the mmf of the stator. Although this expression would be conceptually advantageous, it would be of limited usefulness because an additional unknown—the number of winding turns—is introduced.

5.1.7 Phasor Diagrams

A graphical representation, such as a phasor diagram, often simplifies the solution of a problem and enhances the understanding of the subject matter under consideration. Note, for example, the following equation, which is obtained by applying KVL to the motor's equivalent circuit of Fig. 5-7(b):

$$V_f = \angle -\delta = V_t \angle 0^\circ - I_a (R_a + jX_s) \tag{5.12}$$

Usually, the excitation voltage and its phase angle are unknown. You may mathematically determine the values of the unknowns, but you will often find the graphical solution to be simpler. The graphical solution has the additional advantage of giving a pictorial representation of the various terms and concepts under consideration. This section explains how to draw the phasor diagrams of Eqs. (5.11) and (5.12). These equations are central to understanding the motor's operation.

Figures 5-10(a), (b), and (c) show the phasor diagrams of a synchronous motor operating at unity, at leading, and at lagging power factors, respectively. The sequence of steps required to complete the diagrams is outlined below and is identified on the phasor diagrams by the encircled numbers.

1. Draw the terminal voltage V_t to scale and take it as reference. Recall that for a star-connected motor,

$$V_t = \frac{V_{\text{L-L}}}{\sqrt{3}} \qquad \text{and} \qquad I_L = I_a$$

For a delta-connected motor,

$$V_t = V_{\text{L-L}} \qquad \text{and} \qquad I_L = \sqrt{3} I_a$$

Only the magnitudes of the various parameters are considered here. The subscript L indicates line quantities.

2. Calculate and then draw to scale the armature current I_a. The per-phase armature current is usually obtained from the rating and efficiency of the machine, as the following standard relationships indicate:

$$\text{input power} = \frac{\text{output power}}{\text{efficiency}} \tag{5.13}$$

Using mathematical symbols,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} \tag{5.14}$$

$$P_{\text{in}} = \sqrt{3}\, V_{\text{L-L}} I_L \cos\theta \tag{5.15}$$

where η and $\cos\theta$ represent the motor's efficiency and its power factor, respectively.

Note that the output power is also called brake horsepower, nameplate power, or rated power.

3. From the tip of V_t, draw the phasor $I_a R_a$. Because this represents the voltage drop in the stator resistance, it should be in phase with the armature current. However, owing to the negative sign in Eq. (5.12), it must be placed at an angle 180° from its normal direction.

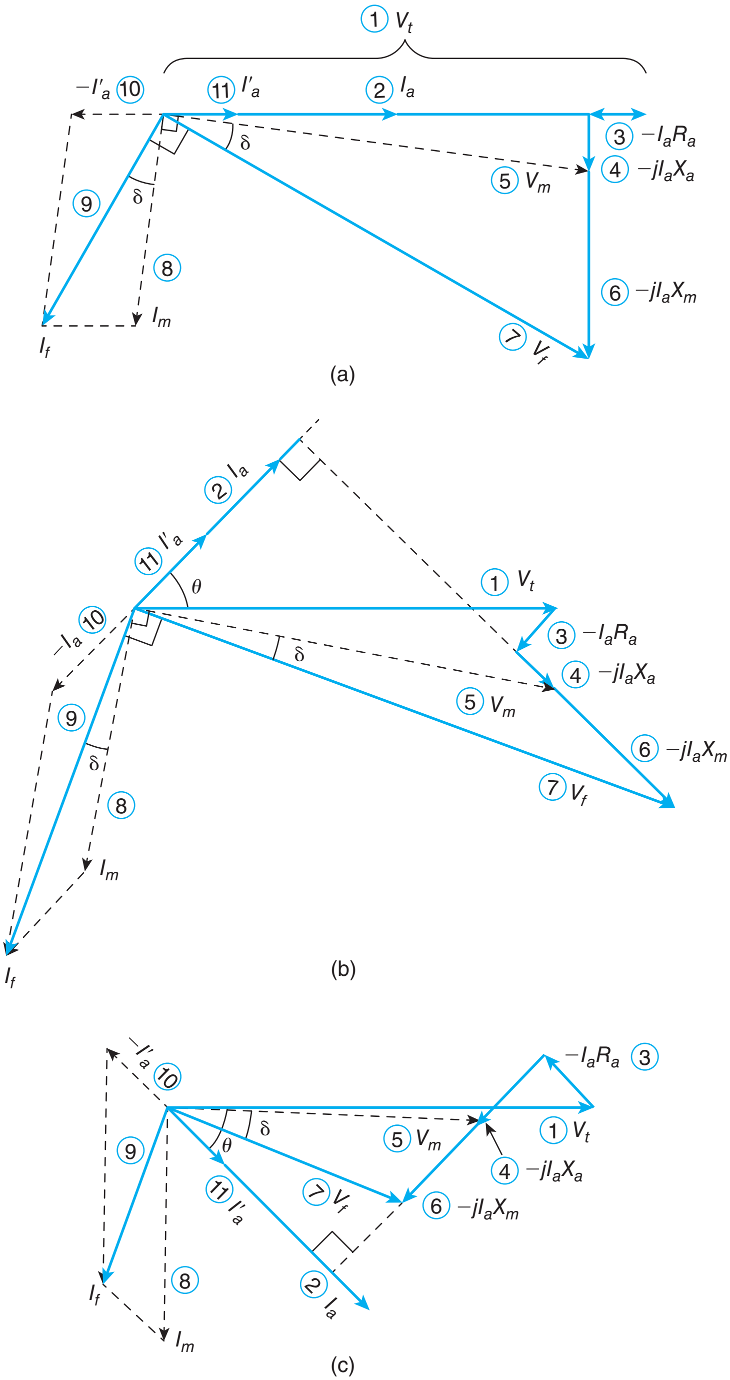


FIG. 5-10 Phasor diagrams for synchronous motors: **(a)** unity power factor, **(b)** leading power factor, **(c)** lagging power factor.

4. From the tip of I_aR_a draw the phasor jI_aX_a. Because this represents the voltage drop in an inductor, it leads the current by 90°. However, the actual position of the phasor jI_aX_a is drawn at an angle 180° from its normal direction, owing to the negative sign of the voltage equation.
5. The tip of the phasor jI_aX_a locates the magnetization voltage V_m.
6. The phasor jI_aX_m is drawn at the extension of jI_aX_a.
7. The line drawn from the origin to the tip of the phasor jI_aX_m gives the excitation voltage V_f.
8. Having calculated V_m, obtain from the open-circuit characteristic the magnetizing component I_m of the field current.
9. Draw the field current I_f at 90° V_f. The magnitude of this current is obtained from the motor's OCC by projecting on it the excitation voltage (see Fig. 5-9).
10. Obtain graphically the reverse of I'_a using Eq. (5.11).
11. Draw I'_a in phase with the armature current.

Comments

As can be seen from Fig. 5-10(c), when the motor operates at a lagging power factor, the field current is smaller than its magnetizing component because of the magnetizing effects of the armature current. Conversely, when the motor operates at a leading power factor, the field current is larger than its magnetizing component because of the demagnetizing effects of the armature current.

Example 5-1

A 50 kW, 480 V, 60 Hz, 900 r/min, 0.8 Pf (power factor) leading, 0.93 efficient, star-connected synchronous motor has a synchronous impedance of 0.074 + j0.48 ohms per phase. When the motor draws rated current, determine:

a. The excitation voltage.
b. The torque angle.

SOLUTION

a. The motor's rated armature current is

$$I_a = \frac{P_{\text{out}}}{\eta\sqrt{3}\,V_{\text{L-L}}\cos\theta}$$

$$= \frac{50{,}000}{0.93\sqrt{3}(480)(0.8)} = 80.83\underline{/36.9^\circ}\text{ A}$$

Applying KVL to the equivalent circuit of the motor (Fig. 5-11(a)), we obtain

$$V_f\underline{/-\delta} = V_t\underline{/0^\circ} - I_a(R_a + jX_s)$$

$$= \frac{480}{\sqrt{3}} - 80.83\underline{/36.9^\circ}\,(0.074 + j0.48)$$

$$= 295.62 - j34.63$$

$$= 297.64 \angle -6.7^\circ \text{ V/phase}$$

and

$$V_f = 515.54 \text{ V, L-L}$$

b. The torque angle is

$$\underline{\underline{\delta = -6.7^\circ}}$$

The phasor diagram is shown in Fig. 5-11(b).

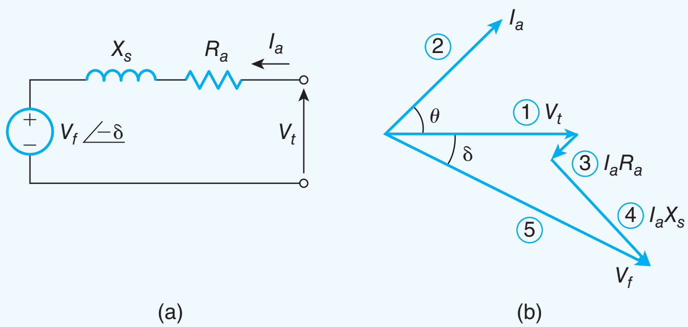


FIG. 5-11

EXAMPLE 5-2

A 480 V, 3-ϕ, 60 Hz, 150 kW, 0.94 efficient synchronous motor has a per-phase synchronous impedance of 0.05 + j0.75 ohms. The leakage reactance is 0.25 ohm per phase. The OCC in the operating region is given by

$$V_f = 10 + 20I_f$$

When the motor draws rated current at unity power factor, determine:

a. The approximate and exact values of the torque angle.
b. The magnetizing component of the field current.
c. The field current.
d. The armature reaction in equivalent field amperes.
e. The effective turns ratio.

SOLUTION

a. The rated current is

$$I_a = \frac{P_{\text{in}}}{\sqrt{3}V_{\text{L-L}}\cos\theta} = \frac{150{,}000}{0.94}\left(\frac{1}{\sqrt{3}(480)(1)}\right) = 191.94\underline{/0^\circ}\ \text{A}$$

The magnetizing voltage is found by applying KVL to the equivalent circuit of Fig. 5-12.

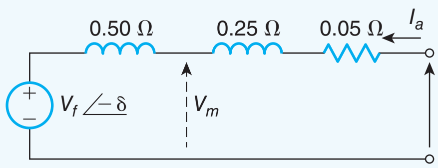


FIG. 5-12

$$V_m = V_t - I_a(R_a + jX_a)$$

$$= \frac{480}{\sqrt{3}}\underline{/0^\circ} - 191.94\underline{/0^\circ}(0.05 + j0.25)$$

$$= 271.8\underline{/-10.2^\circ}\ \text{V/phase}$$

Similarly, the excitation voltage is

$$V_f = \frac{480}{\sqrt{3}}\underline{/0^\circ} - 191.94\underline{/0^\circ}(0.05 + j0.75)$$

$$= 303.8\underline{/-28.3^\circ}\ \text{V/phase}$$

Thus, the approximate and exact values of the torque angle in electrical degrees are

$$\delta_{\text{appr}} = \underline{\underline{-28.3^\circ}}$$

$$\delta_{ex} = -28.3^\circ - (-10.2^\circ) = \underline{\underline{-18.1^\circ}}$$

b. The magnitude of the magnetizing component of the field current can be found from the given open-circuit characteristic:

$$I_m = \frac{V_m - 10}{20} = \frac{271.8\sqrt{3} - 10}{20} = 23.04\ \text{A}$$

This current lags its corresponding voltages by 90°. Thus,

$$I_m = \underline{\underline{23.04}}\underline{/-90^\circ - 10.2^\circ}$$

$$= \underline{\underline{23.04\underline{/-100.2^\circ}\ \text{A}}}$$

c. Similarly, the magnitude of the field current is

$$I_f = \frac{303.8\sqrt{3} - 10}{20} = 25.81 \text{ A}$$

In phasor form:

$$I_f = 25.81 \underline{/-90° - 28.3°}$$

$$= \underline{\underline{25.81 \underline{/-118.3°} \text{ A}}}$$

d. The armature reaction is found by using Eq. (5.11).

$$I'_a = I_m - I_f = 23.04 \underline{/-100.2°} - 25.81\underline{/-118.3°}$$

$$= \underline{\underline{8.16 \text{ A}}}$$

e. The effective turns ratio is obtained from Eq. (5.9).

$$N_e = \frac{I_m}{I'_a} = \frac{191.94}{8.16} = \underline{\underline{23.51}}$$

Exercise 5-1

Prove the following:

a. The torque angle is equal to the phase angle between the field current and its magnetizing component.
b. The approximate value of the torque angle of a synchronous motor that operates at a leading power factor of θ degrees and has negligible armature resistance is given by

$$\delta = \arctan \frac{I_a X_s \cos\theta}{V_t + I_a X_s \sin\theta}$$

c. The per-unit copper losses in a motor are equal to the motor's per-phase armature resistance expressed in per unit.

Exercise 5-2

A 375 kW, 2200 V, 60 Hz, 0.80 Pf lagging, 0.966 efficient, 900 r/min, star-connected motor has a synchronous impedance of $0.015 + j0.702$ pu. When the motor draws rated current at a nominal power factor, determine:

a. The excitation voltage.
b. The approximate torque angle.

Answer (a) 1741.5 V, L-L; (b) $-44.3°$

Exercise 5-3

A synchronous motor has negligible armature impedance. Under rated operating conditions, its power factor is unity and its field current is 30 A. On an open-circuit test, 26 A are required to give the rated terminal voltage. Determine:

a. The armature reaction in equivalent field amperes.
b. The torque angle.

Answer (a) 1497 A, (b) $-29.9°$

5.1.8 Power and Torque Developed

In this section, general expressions will be derived for the complex, real, and reactive powers drawn by a synchronous motor that is connected to an infinite bus (see Fig. 5-13(a)). The system's per-phase equivalent circuit is shown in Fig. 5-13(b). The impedance (Z) is equal to the synchronous impedance of the motor plus the impedance of the transmission line between the motor and the infinite bus.

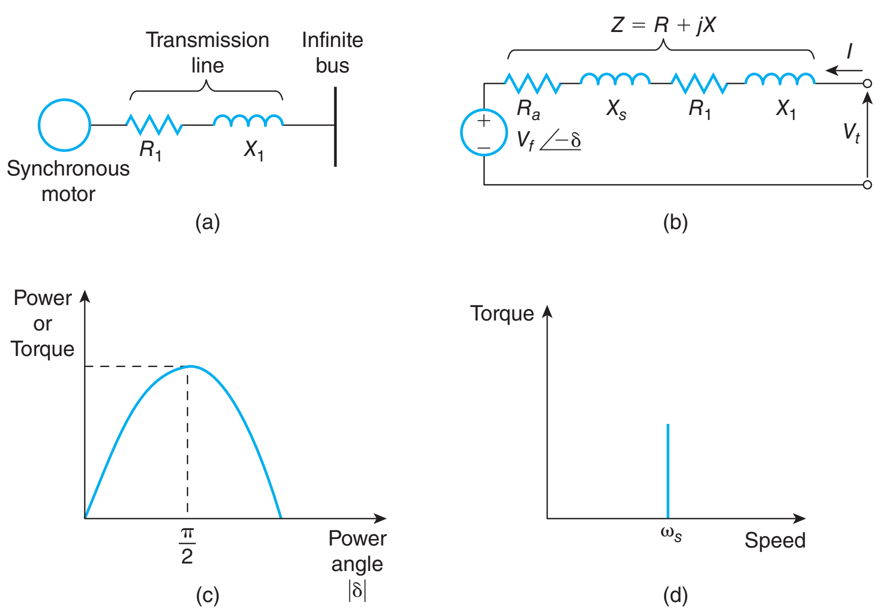


FIG. 5-13 **(a)** Synchronous motor connected to an infinite bus, **(b)** per-phase equivalent circuit, **(c)** power or torque versus load angle, **(d)** torque versus speed.

The complex power drawn from the infinite bus is

$$S = P + jQ = V_t I^* \tag{5.16}$$

where I^* is the conjugate of the current phasor and V_t is now the infinite-bus voltage. Using Ohm's law,

$$I^* = \left(\frac{V_t \underline{/0^\circ} - V_f \underline{/-\delta}}{Z} \right)^* \tag{5.17}$$

By definition,

$$Z = |Z| \underline{/\theta} \tag{5.18}$$

From Eqs. (5.16), (5.17), and (5.18), we obtain

$$S = \frac{V_t}{|Z|} (V_t \underline{/-\theta} - V_f \underline{/-\theta - \delta})^* \tag{5.19}$$

$$= \frac{V_t}{|Z|} [V_t \cos\theta - j\sin\theta) - V_f (\cos(-\theta - \delta) + j\sin(-\theta - \delta))]^* \tag{5.20}$$

$$= \frac{V_t}{|Z|} [V_t \cos\theta - V_f \cos(-\theta - \delta) - j(V_t \sin\theta + V_f \sin(-\theta - \delta))]^* \tag{5.21}$$

Taking the conjugate of the expression within the brackets, we have

$$S = \frac{V_t}{|Z|} [V_t \cos\theta - V_f \cos(-\theta - \delta) + j(V_t \sin\theta + V_f \sin(-\theta - \delta))] \tag{5.22}$$

The active power drawn from the infinite bus is given by the real component of the complex power. Thus,

$$P = \frac{V_t}{|Z|} [V_t \cos\theta - V_f \cos(-\theta - \delta)] \tag{5.23}$$

$$= \frac{V_t}{|Z|} (V_t \cos\theta - V_f (\cos\theta \cos\delta - \sin\theta \sin\delta) \tag{5.24}$$

Also by definition,

$$\cos\theta = \frac{R}{|Z|} \quad \text{and} \quad \sin\theta = \frac{X}{|Z|}$$

Thus,

$$P = \frac{V_t}{|Z|^2} (V_t R - R V_f \cos\delta + X V_f \sin\delta) \text{ watts/phase} \tag{5.25}$$

The torque angle at which the motor draws maximum power is found by applying the calculus theory of minimum and maximum.

$$\frac{\partial P}{\partial \delta} = \frac{V_t V_f}{Z} (R\sin\delta + X\cos\delta) = 0 \tag{5.26}$$

From the above, the torque angle (δ_{m_1}) at which the motor develops maximum power is

$$\delta_{m_1} = -\arctan \frac{X}{R} \tag{5.27}$$

From Eq. (5.25), by neglecting the resistive component of the impedance and considering only magnitudes, we obtain

$$P = \frac{V_t V_f}{X} \sin \delta \text{ watts/phase} \tag{5.28}$$

The power input to the motor will be equal to the power developed. The power developed provides the output power and the rotational losses.

By definition, the torque is

$$T = \frac{\text{power}}{\text{speed}} = \frac{V_t V_f}{\omega_s X} \sin \delta \text{ N} \cdot \text{m/phase} \tag{5.29}$$

For constant excitation, the power is given by

$$P = K \sin \delta \tag{5.30}$$

and the torque by

$$T = K_1 \sin \delta \tag{5.31}$$

The constants K and K_1 can be obtained from Eqs. (5.28) and (5.29), respectively.

As shown in Fig. 5-13(c), the maximum torque of the motor occurs at a power angle of $\pi/2$ radians. Maximum torque is often referred to as breakdown torque or pull-out torque. Since the speed of the motor is constant, the torque-versus-speed characteristic will be as shown in Fig. 5-13(d).

The imaginary part of the complex power of Eq. (5.22) gives the reactive power drawn from the infinite bus. Thus,

$$Q = \frac{V_t}{|Z|}[V_t \sin \theta + V_f \sin(-\theta - \delta)] \tag{5.32}$$

and

$$Q = \frac{V_t}{|Z|}[V_t \sin \theta - V_f(\cos \theta \sin \delta + \sin \theta \cos \delta)] \tag{5.33}$$

Replacing $\sin \theta$ and $\cos \theta$ as before by their equivalent expressions, we get

$$Q = \frac{V_t}{|Z|^2}(XV_t - V_f R \sin \delta - XV_f \cos \delta) \text{ VAR/phase} \tag{5.34}$$

The maximum reactive power drawn from the infinite bus will occur at a torque angle of

$$\delta_{m_2} = \arctan \frac{R}{X} \tag{5.35}$$

By neglecting the resistive component of the impedance in Eq. (5.34), we obtain

$$Q = \frac{V_t}{X}(V_t - V_f \cos \delta) \tag{5.36}$$

When the quantity within the parentheses of Eq. (5.36) is positive, the reactive power is also positive. Thus, the machine operates at a lagging power factor, and the motor is said to be underexcited. Conversely, when the quantity within the parentheses is negative, the motor operates at a leading power factor, and the motor is said to be overexcited.

When the parameters of Eqs. (5.25), (5.28), (5.34), and (5.36) are expressed in per unit, then the derived formulas give the corresponding total output of the machine in per unit. When a synchronous motor is used only to improve the power factor of a network, its torque angle is approximately zero. Thus, the reactive power drawn by the motor is

$$Q = \frac{V_t(V_t - V_f)}{X_s} \text{ VAR/phase} \tag{5.37}$$

Equation (5.37) makes it clear that, by varying the dc current, the reactive power of a synchronous motor can be varied smoothly in either direction. In comparison, the reactive power of a capacitor can change only in steps (discontinuously) and only in one direction.

Figure 5-14(a) shows a synchronous motor used to improve the power factor of a load. The phasor diagram of the pertinent parameters is shown in Fig. 5-14(b).

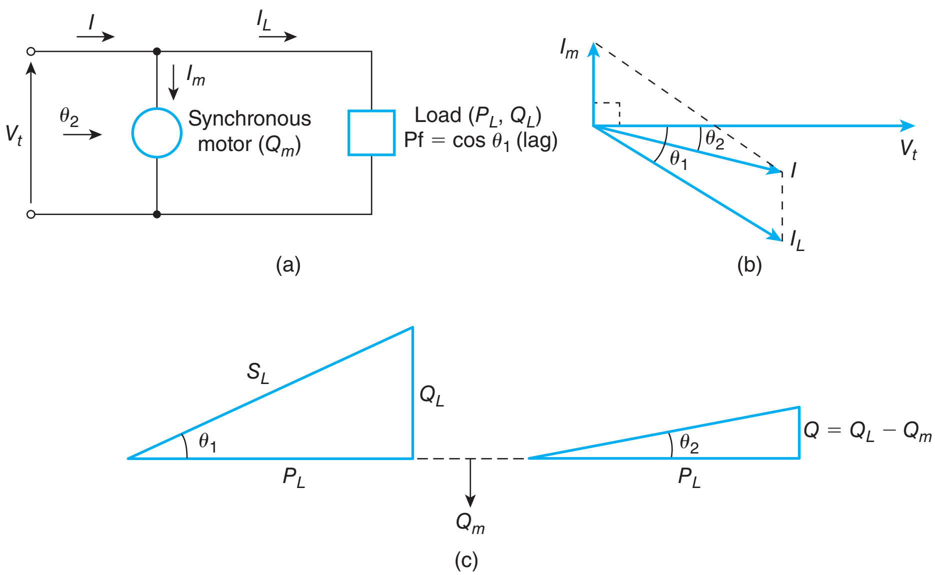


FIG. 5-14 Ideal synchronous motor and its effects on the overall power factor of a plant: **(a)** electrical network, **(b)** phasor diagram, **(c)** power triangles of the load, motor, and source.

The power triangles of the load, motor, and source are shown in Fig. 5-14(c). Note that, in this case, a synchronous motor draws only reactive power.

Speed Control

The speed of a synchronous motor can be controlled either by changing the number of poles or by varying the frequency of the stator voltage. The frequency is varied through "frequency converters," and the number of poles is changed by reconnecting the stator and rotor windings into the desired pole configuration. The technique of frequency conversion is the same for both synchronous and three-phase induction machines, and so it need not be discussed again here. One exercise will be given, however, just to review the essentials.

5.1.9 Effects of Field Current on the Characteristics of the Motor

Previous sections discussed the effects of the field current on the motor's internal parameters, such as torque angle, excitation voltage, and the magnetization of the machine. This section will analyze the effects of the field current on the motor's external parameters. Specifically, qualitative proofs will be developed that give the power factor, the armature current, and the real and reactive powers as functions of the field current. The qualitative proofs are based on two assumptions: that the motor's output power remains constant, and that its stator resistance is negligible. These proofs will lead to the construction of the motor's external characteristics. One can easily predict these characteristics by first drawing a phasor diagram at unity power factor and observing the changes in the various parameters that accompany the increase or decrease of excitation voltage.

The curves of the power factor and of the armature current as a function of the field current resemble the letter V and are often referred to as the Vee curves of synchronous machines. From a practical point of view, these curves constitute one of the motor's most important characteristics.

Power Factor versus Field Current

The power-factor versus field-current characteristic of the motor is derived as follows:

1. The phasor diagram of a synchronous motor operating at unity power factor is shown in Fig. 5-15(a). The required field current for this operating condition corresponds to what is referred to as 100% excitation.
2. By increasing the field current above its normal value (Fig. 5-15(b)), the excitation voltage (V_{f_2}) will also be increased. From Eq. (5.28), it is clear that the torque angle δ_2 will be smaller than the torque angle. From the tip of the phasor V_t to the tip of the new excitation voltage V_{f_2} will be the phasor $I_{a_2} X_s$. This is so because of the following general voltage equation:

$$V_{f_2} = V_t - jI_{a_2} X_s \tag{5.38}$$

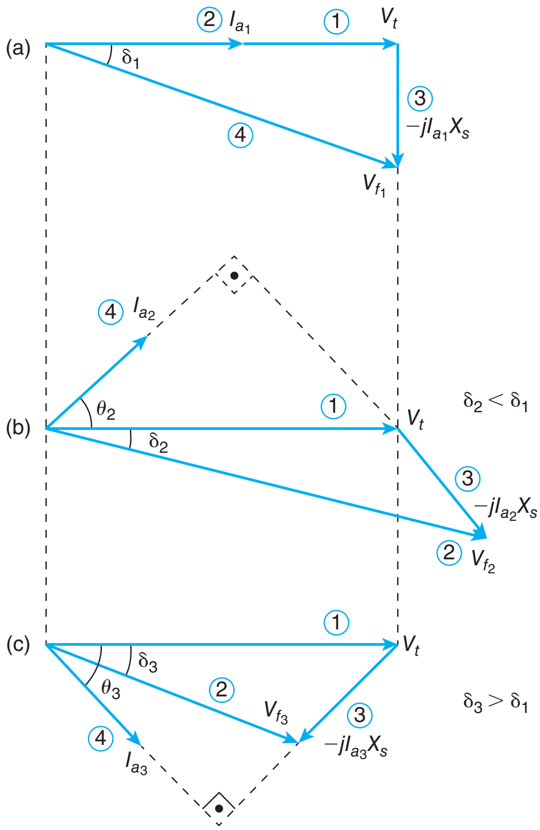


FIG. 5-15 Phasor diagrams for the qualitative derivation of Vee curves: **(a)** unity power factor (normal excitation), **(b)** leading power factor (overexcitation), and **(c)** lagging power factor (underexcitation).

The extension of $I_{a_2}X_s$ (dotted line) must cross the line that represents the armature current at 90°. This is possible only when the armature current *leads* the terminal voltage, as shown in Fig. 5-15(b).

3. By decreasing the field current below its normal value (Fig. 5-15(c)), the excitation voltage (V_{f3}) will also be decreased, and the new torque angle δ_3 will be larger than the torque angle δ_1. As before, from the tip of the phasor V_t to the tip of the new excitation voltage will be the phasor $I_{a_3}X_s$. The dotted extension of the phasor $I_{a_3}X_s$ must cross the line of the armature current at 90°. This is possible only when the armature current *lags* the terminal voltage, as shown in Fig. 5-15(c)).

As this discussion makes clear, a change in the field current will be accompanied by a change in the power factor of the motor. Qualitatively speaking, the

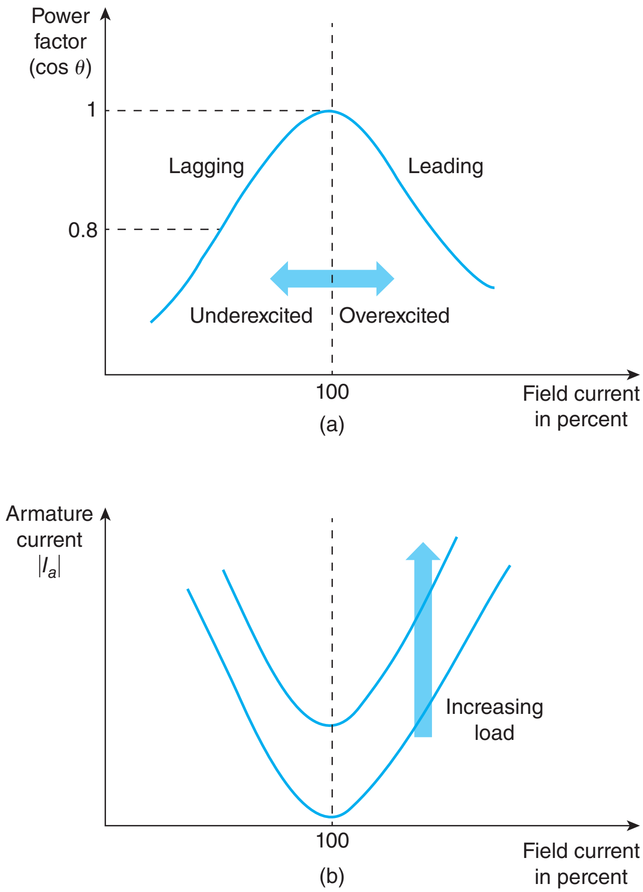


FIG. 5-16 Vee curves for a synchronous motor: **(a)** power factor versus field current, **(b)** magnitude of armature current versus field current.

variation of the power factor as a function of the field current can be represented by a curve that resembles an inverted V, as shown in Fig. 5-16(a).

Magnitude of Armature Current versus Field Current

Neglecting armature resistance, we find that the input power is equal to the power developed, which is assumed to remain constant. Mathematically,

$$V_t I_a \cos\theta = P_d \text{ watts/phase} \tag{5.39}$$

From Eq. (5.39),

$$I_a = \frac{K}{\cos\theta} \tag{5.40}$$

where the constant K is given by

$$K = \frac{P_d}{V_t} \tag{5.41}$$

Qualitatively speaking, the previously derived Vee curves of Fig. 5-16(a) indicate the following:

For variations of the field current up to 100% excitation,*

$$\cos\theta \propto I_f \tag{5.42}$$

For variations of the field current above 100% excitation,

$$\cos\theta \propto \frac{1}{I_f} \tag{5.43}$$

From Eqs. (5.40), (5.42), and (5.43), we obtain

$$\text{For excitation above 100\%,} \qquad I_a \propto I_f \tag{5.44}$$

$$\text{For excitation below 100\%,} \qquad I_a \propto \frac{1}{I_f} \tag{5.45}$$

The variation of the armature current as a function of field current may be represented by the curves shown in Fig. 5-16(b), whose shape resembles the letter V. The armature current is minimum when the power factor is maximum. In other words, minimum armature current corresponds to 100% excitation or unity power factor.

Reactive Power versus Field Current

By definition, the reactive power drawn by a motor is

$$Q = V_t I_a \sin\theta \ \text{ VAR/phase} \tag{5.46}$$

The Vee curves of Fig. 5-16(a) indicate that for excitation up to 100%,

$$\cos\theta \propto I_f$$

An increase in the cosine function will be accompanied by a decrease in the sine function. Thus,

$$\sin\theta \propto \frac{1}{I_f} \tag{5.47}$$

*The proportionality symbols indicate the trend in the change of the variables but not true direct proportion or inverse proportion.

From Eqs. (5.45), (5.46), and (5.47), we get

$$Q \propto \frac{1}{I_f^2} \tag{5.48}$$

Similarly, for excitation above 100%, from Eqs. (5.43), (5.44), and (5.46), we obtain

$$Q \propto I_f^2 \tag{5.49}$$

From Eqs. (5.48) and (5.49), the reactive power versus the motor's field current can be drawn as shown in Fig. 5-17. The leading power factor corresponds to positive VAR and the lagging power factor to negative VAR. At 100% field excitation, the power factor is equal to unity and the reactive power is equal to zero.

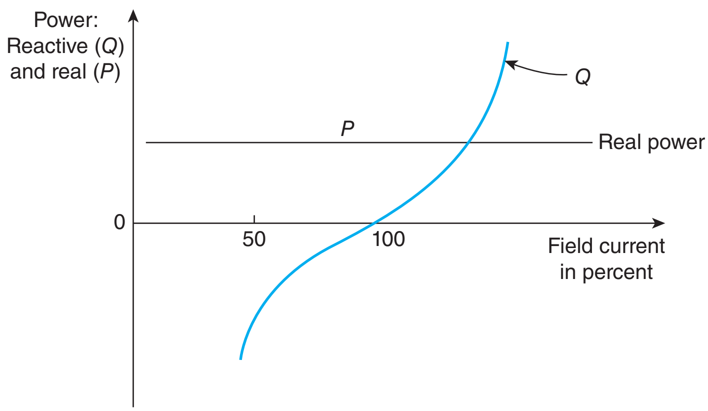


FIG. 5-17 A synchronous motor's real and reactive powers as a function of field current.

Real Power

The real power drawn by a synchronous motor is independent of the field current because the real component of the armature current ($I_a \cos \theta$) does not depend on the excitation. The variation of the real power as a function of the field current is shown in Fig. 5-17. In an actual machine, however, the stator ($I_a^2 R_a$) losses, and hence the input power, will increase whenever the stator current increases as a result of changing the field current.

The curves derived in this section are approximate and are to be used for qualitative analysis only. Exact relationships can be derived from the magnetization characteristic of the particular motor and by use of the appropriate equations discussed in this chapter.

EXAMPLE 5-3

Explain:

a. The fact that the torque angle of a synchronous motor always lags the terminal voltage.
b. The fact that a sudden increase in the torque requirements of a synchronous motor will momentarily decrease the synchronous speed of the rotor, and thus the torque angle will be enlarged.
c. The magnetizing and demagnetizing effects of armature current. Use phasor diagrams to illustrate the concepts.

SOLUTION

a. The output power of a synchronous motor is produced by the transformation of electrical input power. In other words, before the motor starts to rotate and produce an output, the electrical input (V_t) must first be applied; the rotation of the shaft will then follow (V_f). Since the torque angle is approximately equal to the phase angle between the terminal voltage and the excitation voltage, the torque angle is negative, or lags the applied voltage V_t, as shown in Fig. 5-18. The excitation voltage is produced by the flux of the rotor, which rotates at the same angular speed as the shaft.

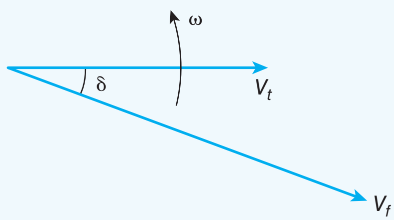


FIG. 5-18

b. As shown by the following equation, a sudden increase in the load torque requirements of a synchronous motor will be accompanied by an increase in the torque angle:

$$P = V_f \frac{V_t}{X_s} \sin \delta$$

Since the torque angle is always lagging, its enlargement will force the shaft to momentarily rotate at a lower speed relative to the stator field. Alternatively, owing to physical considerations, loading the shaft of the motor will momentarily reduce the rotor's angular speed and thus, as depicted in Fig. 5-18, the torque angle will become larger. This phenomenon is only transient. Once the power equilibrium is reestablished, the rotor will attain synchronous speed and the torque angle will settle at a magnitude that will satisfy the new power requirement.

c. Economic considerations dictate that, under normal operating conditions, the motor should operate at a point close to the knee of the magnetization curve. Achieving this level of magnetization requires a certain amount of field current. Since the field current has two components, its magnitude will depend on the relative magnitudes and positions of its components. Rewriting the equation for the field current, we have

$$I_f = I_m - I'_a$$

When the motor operates at a lagging power factor, as shown in Fig. 5-10(c), less field current is required than when it operates at a leading power factor, as shown in Fig. 5-10(b). At a lagging power factor, on the one hand, the flux of the armature current aids the magnetizing effect of the field current. This is known as the *magnetizing effects of the armature current*. At a leading power factor, on the other hand, the armature current produces flux that opposes the magnetization of the machine. This is referred to as the *demagnetizing effects of the armature current.*

Example 5-4

1. Define power factor and give five advantages that result from its improvement.
Why are synchronous condensers preferred over static capacitors?

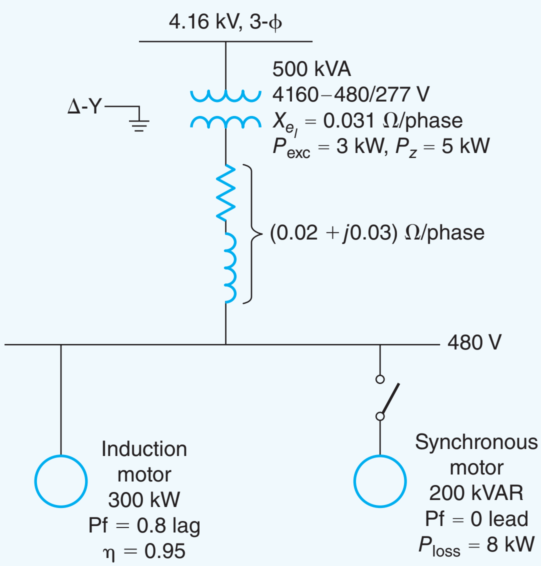


FIG. 5-19(a)

2. For the network shown in Fig. 5-19(a), calculate with and without the synchronous motor in the circuit:
   a. The losses in the feeder.
   b. The kVA delivered by the transformer.
   c. The losses in the transformer.
   d. The regulation.

SOLUTION

1. The power factor of a given circuit is equal to the cosine of the phase angle between the circuit voltage and current. The voltage is always taken as reference.

   The ordinary consumer of electrical energy is not interested in the phase angle between the voltage and the current applied to a motor. He or she is concerned not with power factor, but with the actual kW output of his motor. However, for constant voltage and output power, the current through the incoming feeders depends on the power factor, as shown by the following equation:

$$I = \frac{P_{\text{in}}}{\sqrt{3}V_{L\text{-}L}\cos\theta}$$

   For example, a reduction in the power factor from unity to 0.80 will result in a 25% increase in the current. Conversely, an increase (improvement) in the power factor will result in decreasing current, which will reduce the following:
   a. The losses in the upstream feeders.
   b. The required size of the upstream feeders.
   c. The required size of the transformer.
   d. The losses in the transformer.
   e. The voltage fluctuation across the motors and other electrical loads.

   Synchronous motors can provide output torque and leading or lagging kVAR to the network. Their operation is very smooth and their losses are nominal. Capacitors can provide only kVAR lagging in steps, but they consume negligible power. For simplicity, we assume here that the motor does not provide any torque.

   Furthermore, the effects of harmonics and switching problems are more severe for capacitors than for synchronous motors. Nevertheless, a synchronous machine has maintenance requirements for its bearings, ventilation, and so on, that static capacitors do not have.

2. *Without the synchronous motor in the circuit*:
   a. The line or feeder current is

$$I = \frac{300{,}000}{0.95\sqrt{3}(0.8)480} = 474.79\,\underline{/-36.9^\circ}\text{ A}$$

Thus, the losses in the feeder (P_{lf}) are

$$P_{lf} = 3(0.02)(474.79)^2 = \underline{\underline{13.53 \text{ kW}}}$$

b. The transformer should supply the power required by the load plus the power consumed by the feeder. From the circuit of Fig. 5-19(b), the voltage

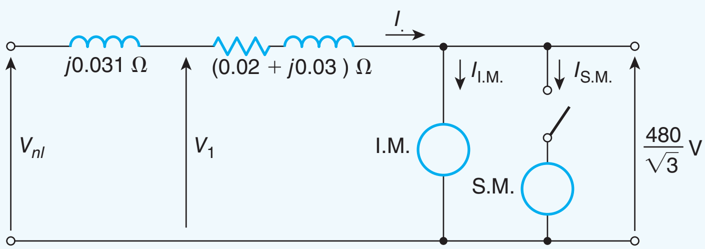


FIG. 5-19(b)

at the output terminals of the transformer is

$$V_1 = V_L + IZ = \frac{480}{\sqrt{3}}\underline{/0^\circ} + 474.79\underline{/-36.9^\circ}(0.02 + j0.03)$$

$$= 293.32\underline{/-1.1^\circ} \text{ V/phase,} \quad \text{and} \quad V_1 = 508.1 \text{ V L-L}$$

Thus, the apparent power delivered by the transformer is

$$S = \sqrt{3}\, V_{\text{L-L}} I_L$$

$$= \sqrt{3}(508.1)(474.79) = \underline{\underline{417.8 \text{ kVA}}}$$

c. Assuming that the core loss is proportional to the square of the terminal voltage, the transformer's losses are

$$P_{l_{(t)}} = 3\left(\frac{508.1}{480}\right)^2 + 5\left(\frac{474.79}{601.4}\right)^2 = \underline{\underline{6.48 \text{ kW}}}$$

d. The no-load voltage at the secondary of the transformer is

$$V_{nl} = \frac{480}{\sqrt{3}}\underline{/0^\circ} + 474.79\underline{/-36.9^\circ}(0.02 + j0.03 + j0.031)$$

$$= 302.61\underline{/-3.3^\circ} \text{ V/phase}$$

and

$$V = (\sqrt{3}(302.61) = 524.13 \text{ V, L-L}$$

$$R\% = \frac{524.13 - 480}{480}(100) = \underline{\underline{9.19\%}}$$

3. *With the synchronous motor in the circuit*:

a. The current drawn by the synchronous condenser is found as follows:

$$Q = \sqrt{3}\, V_{\text{L-L}} I_L \sin\theta = 200 \text{ kVAR}$$

$$P = \sqrt{3}\, \text{V}_{\text{L-L}} I_L \cos\theta = 8 \text{ kW}$$

From the above, the current drawn by the synchronous motor is

$$I_{\text{S.M.}} = 240.76 \angle 87.7^\circ \text{ A}$$

The current in the feeder is the sum of the currents drawn by the induction and synchronous motors. Thus,

$$I_f = 474.79\angle -36.9^\circ + 240.76\angle 87.7^\circ$$

$$= 391.97\angle -6.5^\circ \text{ A}$$

The new power factor is

$$\cos 6.5^\circ = 0.99 \text{ lag}$$

The losses in the feeder are

$$P_{lf} = 3(0.02)(391.97)^2 = \underline{\underline{9.22 \text{ kW}}}$$

b. The voltage at the output of the transformer is

$$V_1 = \frac{480}{\sqrt{3}}\angle 0^\circ + 391.97\angle -6.5^\circ (0.02 + j0.03)$$

$$= 286.15\angle -2.2^\circ \text{ V/phase}$$

Then

$$V_1 = \sqrt{3}(286.15) = 496.15 \text{ V, L-L}$$

The kVA delivered by the transformer is

$$S = \sqrt{3}(496.15)(391.97) = \underline{\underline{336.84 \text{ kVA}}}$$

c. The transformer losses are

$$P_{l_{(t)}} = 3\left(\frac{496.15}{480}\right)^2 + 5\left(\frac{391.97}{601.4}\right)^2 = \underline{\underline{5.33 \text{ kW}}}$$

d. The no-load voltage at the substation is

$$V_{nl} = \frac{480\angle 0^\circ}{\sqrt{3}} + 391.97\angle -6.5^\circ(0.02 + j0.03 + j0.031)$$
$$= 288.53\angle 4.55^\circ \text{ V/phase}$$

Then

$$V_{nl} = \sqrt{3}(288.53) = 499.75 \text{ V, L-L}$$

The voltage regulation is

$$R\% = \frac{499.75 - 480}{480}(100) = \underline{\underline{4.1\%}}$$

For purposes of comparison, the results are summarized in Table 5-2.*

TABLE 5-2 Summary of results of Example 5-4

Synchronous Motor	*Power Factor at the 480 V Terminals*	*kVA Delivered by the Transformer*	*Losses in the Transformer in kW*	*Losses in the Feeder in kW*	*Current through the Feeder in A*	*Voltage Regulation in Percent*
Disconnected	0.8 lagging	417.8	6.48	13.53	474.79	9.19
Connected	0.99 lagging	336.84	5.33	9.22	391.97	4.1

The advantages of higher-power-factor operation become very evident when we compare the tabulated results. The adverse effects of low-power-factor operation of large industrial customers on the upstream equipment cannot be tolerated.

*The student may evaluate some of the financial advantages of the high-power-factor operation by assuming the following:

- Cost of energy: $1000/kW/year. Cost of transformers: $50/kVA.
- Cost of a three-conductor cable: $30/meter per ampere capacity.
- Length of cable: 200 meters.
- Monthly power-factor penalty:

$$\$10 \text{ (actual kW)} \left(\frac{\text{Pf}_n}{\text{Pf}_a} - 1\right)$$

where
Pf_a = actual power factor
Pf_n = minimum permissible power factor, usually 0.90

- The cost of a 480 V capacitor bank is about $40/kVAR.

Exercise 5-4

a. A synchronous motor, when operated at 0.8 Pf leading, has an armature copper loss of 4 kW. *Estimate* the armature copper loss when the motor operates at unity power factor. Assume that the power drawn by the motor remains constant.
b. Derive Eq. (5.37).

Answer (a) 2.56 kW.

Exercise 5-5

The speed of a 480 V, 60 Hz, 10-pole, 0.94 efficient, unity-power-factor synchronous motor is controlled through a frequency converter. The synchronous reactance is 1.04 Ω/phase, and the motor drives a 50 kW constant-power load. When the output voltage of the converter is 432 V, determine:

a. The speed of the motor.
b. The excitation voltage and the torque angle.

Answer **(a)** 648 r/min; **(b)** 256.5 V/phase, −8.6°

5.2 Three-Phase Cylindrical Rotor Machines: Generators

5.2.1 Equivalent-Circuit and Phasor Diagrams

This section deals with the principle of operation, governing equations, characteristics, and the measurement of the parameters of three-phase generators. As mentioned earlier, three-phase generators transform mechanical power into electrical power. The mechanical power of the prime mover rotates the shaft of the generator on which the dc field windings are installed. The speed of the prime mover is maintained at a constant level through electronically controlled speed regulators, commonly known as governors. The rotation of the dc flux cuts the windings of the armature, and, because of the induction principle, a three-phase voltage is generated.

Three-phase generators are also called three-phase alternators. Structurally, they are similar to three-phase synchronous motors.

The direction of energy flow is shown in Fig. 5-20(a). The approximate per-phase equivalent circuit as seen from the stator and rotor terminals is shown in Figs. 5-20(b) and (c), respectively. The physical significance and nomenclature of each parameter are identical to those of three-phase synchronous motors.

The governing equations for alternators are

$$V_f \underline{/\delta^\circ} = V_t \underline{/0^\circ} + I_a (R_a + jX_s) \quad (5.50)$$

$$I_f = I_m + I'_a \quad (5.51)$$

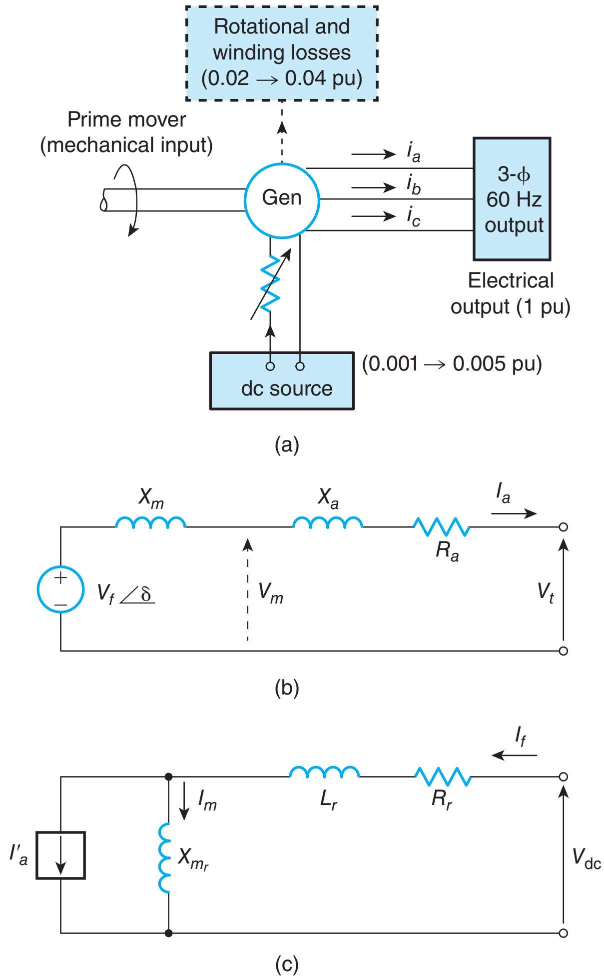


FIG. 5-20 Alternators: **(a)** direction of energy flow and typical losses for a 400 kVA alternator, **(b)** approximate equivalent circuit as seen from the stator, and **(c)** partial equivalent circuit as seen from the rotor.

Note that these equations have a positive sign, whereas those for synchronous motors have a negative sign.

The torque angle of an alternator, owing to physical considerations, always leads the terminal voltage.

The Vee curves for alternators are similar to those for synchronous motors, except that the words "leading" and "lagging" should be switched. For example, an *underexcited* generator (leading Pf) will correspond to an *overexcited* (leading Pf) motor.

The power factor of a single alternator does not depend on its field current but on the impedance of the connected load. However, if an alternator operates in parallel with other generators, their individual power factors can be controlled by properly adjusting their corresponding field currents.

The phasor diagrams for unity and lagging power factor are drawn in Figs. 5-21(a) and (b), respectively. The order of construction is identified by the encircled numbers. The phasor diagrams of generators are easier to draw than those of motors because of the positive sign in the governing equations, Eqs. (5.50) and (5.51).

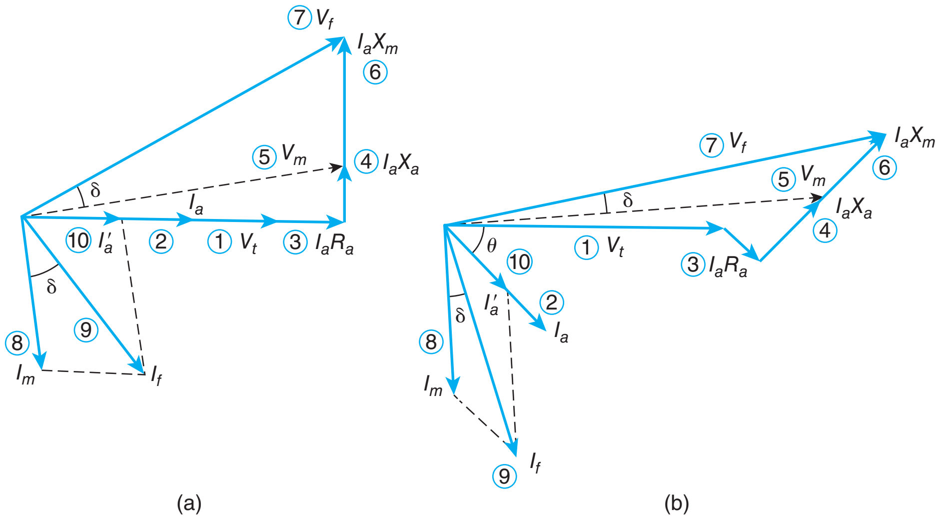


FIG. 5-21 Phasor diagrams for alternators: **(a)** unity power factor, **(b)** lagging power factor.

Notice that at lagging-power-factor operation a large field current is required in order to magnetize the stator and thus to enable the generator to provide rated terminal voltage. In other words, lagging-power-factor currents have a demagnetizing effect on the stator. For this reason, normally a generator *cannot* start when its load is an ac motor of about equal rating. At starting, an ac machine draws a highly inductive current ($\theta \approx 70°$) of a magnitude five to six times larger than the rated value.

Therefore, to select a standby generator, you have to carefully evaluate the nature of the connected load. The normal practice is to consult with the manufacturers regarding the required size and *starting capability* of the unit under consideration. A general rule of thumb is that a generator can start an ac machine provided that the starting kVA of the motor is not more than twice the nominal kVA of the generator. When a properly sized capacitor bank is used, its leading-power-factor currents reduce the magnitude of the downstream inductive currents, thus the generator's starting capability is accordingly increased.

5.2.2 Regulator of Alternators

For practical application, it is important to be able to determine the regulation of a three-phase synchronous generator. Mathematically, the regulation (R), as explained in Chapter 2, is defined by

$$R = \left. \frac{|V_{nl}| - |V_{fl}|}{|V_{fl}|} \right|_{I_f = \text{constant}} \tag{5.52}$$

or

$$R = \left. \frac{|V_f| - |V_{fl}|}{|V_{fl}|} \right|_{I_f = \text{constant}} \tag{5.53}$$

where

V_{nl} = the voltage across the armature terminals at no-load. In the absence of saturation, the no-load voltage is equal to the excitation or internal voltage (V_f). Under this assumption, this voltage can be calculated from Eq. (5.50), or it can be obtained from the Open circuit characteristic once the actual field current is known.

V_{fl} = the voltage across the armature terminals at full-load conditions. This is the rated voltage or the voltage specified on the name-plate of the alternator.

Owing to the effects of armature reaction, the regulation of an alternator often may vary by plus or minus 30%. Thus, the output voltage of a generator rated at 1000 V will fluctuate between 700 and 1300 V, while the excitation remains constant and the armature current varies from its no-load to its full-load value.

The power factor of the load has a substantial effect on the generator's terminal voltage. Nevertheless, proper adjustments of the field current will result in constant output voltage or zero regulation, as shown in Fig. 5-22(a).

Figure 5-22(b) shows that, for a particular value of armature current, capacitive loads increase the terminal voltage while inductive loads decrease it. The

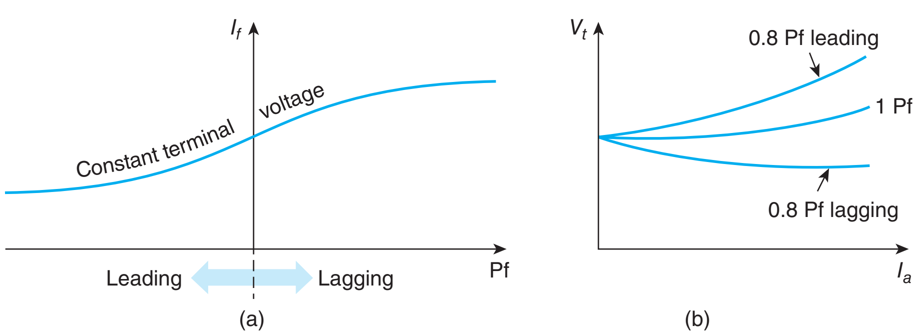


FIG. 5-22 Characteristics of alternators: **(a)** field current versus power factor for constant terminal voltage, **(b)** terminal voltage as a function of armature current for fixed excitation and variable power factor.

reason is that lagging-power-factor armature currents have a demagnetizing effect on the machine, while leading-power-factor currents have a magnetizing effect.

In practice, automatic regulators are used to adjust the field current in such a way as to maintain constant output voltage (zero regulation), independently of load condition.

The regulation of large three-phase alternators cannot be measured directly in the laboratory because it is difficult to find large enough loads to satisfy the required full-load condition. Theoretically, however, a generator's voltage regulation can be determined by the governing equations and from the data on its characteristics. These characteristics are discussed in the following sections.

5.2.3 Characteristics of Alternators

Open-Circuit Characteristic

The open-circuit characteristic (OCC) shows the relationship between the field current and the voltage across the open-circuited terminals of the armature windings. Laboratory data for this characteristic are obtained by driving the machine as a generator at rated synchronous speed (see Fig. 5-23(a)). When the armature and field currents are equal to zero, the power furnished by the prime mover constitutes the mechanical losses of the machine at no-load. A typical OCC is shown in Fig. 5-23(b).

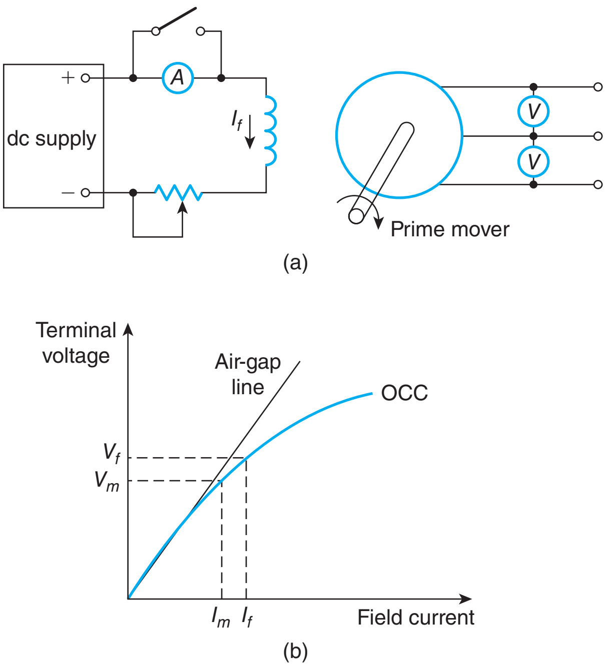


FIG. 5-23 Alternator: **(a)** schematic for the laboratory setup used to obtain data for the magnetization curve, **(b)** typical open-circuit characteristic.

The open-circuit characteristic can be used to obtain the magnetizing component (I_m) and/or the total field current (I_f) once the magnetizing voltage (V_m) and the excitation voltage (V_f) have been calculated from the machine's operating data and equivalent circuit. The open-circuit characteristic is also called the magnetization curve, the no-load characteristic, or the saturation curve.

The line drawn tangentially to the linear portion of the open-circuit characteristic is called the air-gap line. This straight line is the magnetic characteristic of the machine when saturation is neglected.

Short-Circuit Characteristic

The short-circuit characteristic (SCC) shows the relationship between the field current and the armature current when the alternator terminals are shorted. The machine is driven as a generator, and the short-circuit armature current (I_a) is recorded while the field current (I_f) is gradually increased. The power delivered by the prime mover is equal to the rotational losses plus the stator copper losses.

A typical SCC and the laboratory setup that may be used to obtain it are shown in Figs. 5-24(a) and (b), respectively. Since the terminal voltage (V_t) is equal to zero, the excitation voltage, as can be seen from the equivalent circuit of Fig. 5-24(c), is given by

$$V_f = I_a\,(R_a + jX_s) \tag{5.54}$$

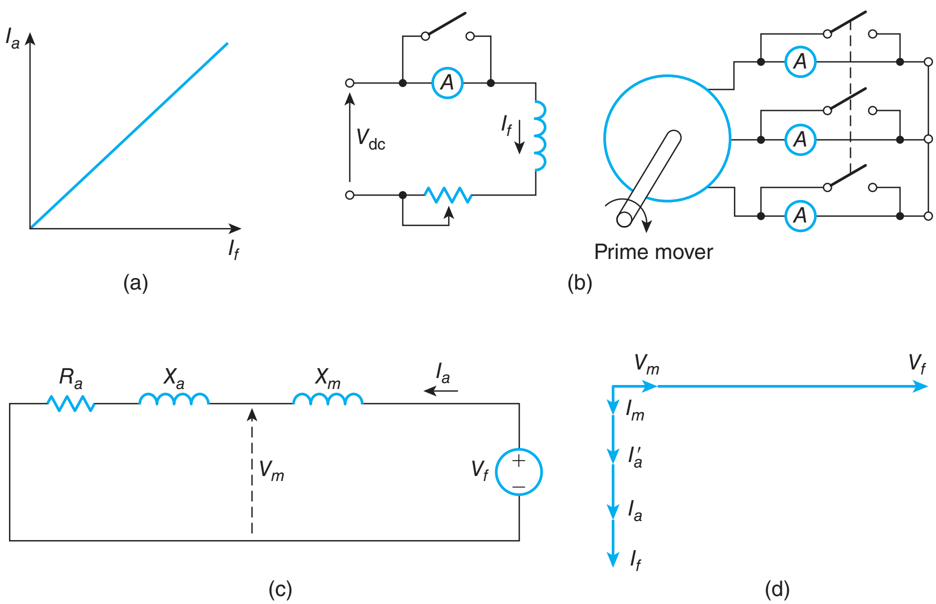


FIG. 5-24 **(a)** Typical short-circuit characteristic, **(b)** schematic for the laboratory setup that may be used to obtain data for the short-circuit characteristic, **(c)** equivalent circuit, **(d)** approximate phasor diagram ($R_a = 0$).

Usually, the armature resistance is negligible. Thus,

$$V_f \approx jI_aX_s \quad \textbf{(5.55)}$$

From the last relationship, it is evident that the armature current lags the excitation voltage by 90°. Rewriting (Eq. 5.51), we have

$$I_f = I_m + I'_a \quad \textbf{(5.56)}$$

I_m is very small because the magnetizing voltage is negligible. I_f also lags V_f by 90°. Therefore, the field current I_f and the armature reaction in equivalent field amperes (I'_a) are approximately in phase. This implies that a linear change in the armature current will be accompanied by a linear change in the field current. The phasor diagram for the short-circuit condition is shown in Fig. 5-24(d).

The short-circuit characteristic is also called the transformation-ratio curve, because its slope gives the approximate value of the effective turns ratio. Thus,

$$\text{slope of SCC} = \frac{I_a}{I_f} = \frac{I_a}{I_m + I'_a} \quad \textbf{(5.57)}$$

Since I_m is negligible,

$$\text{slope of SCC} \approx \frac{I_a}{I'_a} = N_e \quad \textbf{(5.58)}$$

(A more accurate value for this parameter can be obtained from the zero-power-factor characteristic that is discussed in the following section).

By reducing the speed from its rated value, the excitation voltage (V_f) will also be reduced. From the generator principle (see Chapter 6, Section 6.1),

$$V_f = K\phi\omega \quad \textbf{(5.59)}$$

where

K = a constant of the machine.

ϕ = the effective flux.

ω = the angular speed of rotation.

The impedance of the armature circuit, being largely inductive, will also be reduced accordingly.

$$Z_s \approx jX_s \quad \textbf{(5.60)}$$

and

$$Z_s \approx j\omega L_s \quad \textbf{(5.61)}$$

where L_s is the synchronous inductance of the machine. Thus, when the machine is short-circuited, the armature current is given by

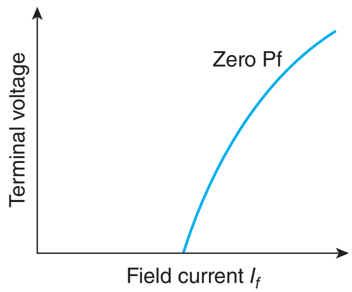
$$I_a = \frac{V_f}{Z_s} \approx \frac{K\phi\omega}{j\omega L_s} = K_1 I_f \quad (5.62)$$

where K_1 is a constant of proportionality. This expression implies that, for a given field current and a moderate variation in the speed of the prime mover, the armature current is constant. A 25% change in the speed of the prime mover above or below its synchronous value does not appreciably affect the linearity of the short-circuit characteristic.

Zero-Power-Factor Lagging Characteristic

The zero-power-factor (ZPF) characteristic shows the relationship between the terminal voltage and the field current at the constant rated armature current (I_a) and zero-load power factor. Lagging loads with a power factor up to 0.1 have a negligible effect on the actual zero-power-factor curve.

A typical ZPF characteristic and a laboratory setup that may be used to obtain it are shown in Figs. 5-25(a) and (b), respectively. The load to the generator



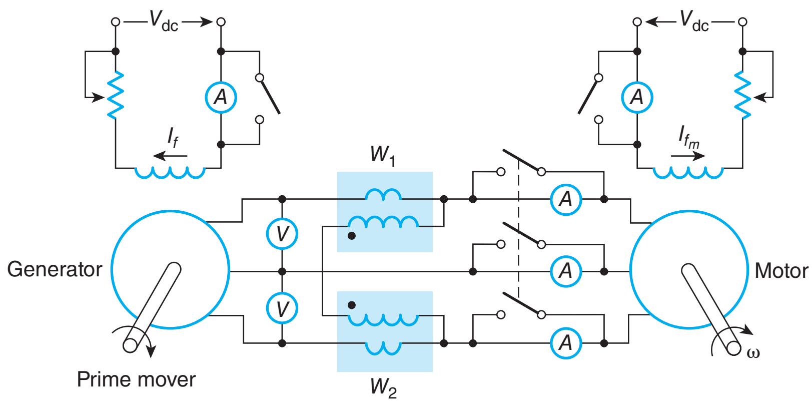


FIG. 5-25 **(a)** Typical lagging zero-power characteristic, **(b)** schematic for a laboratory setup used to obtain data for the zero-power-factor curve.

is a synchronous motor whose power factor and armature current are controlled through its field current. The terminal voltage is controlled by the generator's field current. The output power of the generator must be larger than that of the driven motor because of the motor's losses.

The readings of the instruments on the output terminals of the generator (voltmeters, ammeters, and wattmeters) are also used to calculate the power factor of the load. Instead of the two wattmeters (W_1 and W_2), one may use a polyphase wattmeter. The measurement of three-phase power with two wattmeters is explained in the Appendix (Section A.5).

5.2.4 Measurement of Parameters

Measurement of Synchronous Reactance X_s

The short-circuit characteristic, in conjunction with the magnetization curve, can be used to obtain the unsaturated ($X_{s_{un}}$) and saturated ($X_{s_{sat}}$) values of the synchronous reactance, as follows.

Unsaturated Value

Refer to Fig. 5-26. For any value of field current (I_{f_1}), there is a corresponding voltage on the air-gap line (V_{g_1}) and an armature current (I_{a_1}) on the short-circuit characteristic.

The unsaturated value of the synchronous reactance is given by

$$X_{s_{un}} = \left. \frac{\text{voltage obtained from the air-gap line}}{\text{armature current obtained from the SCC}} \right|_{\text{at any field excitation}}$$

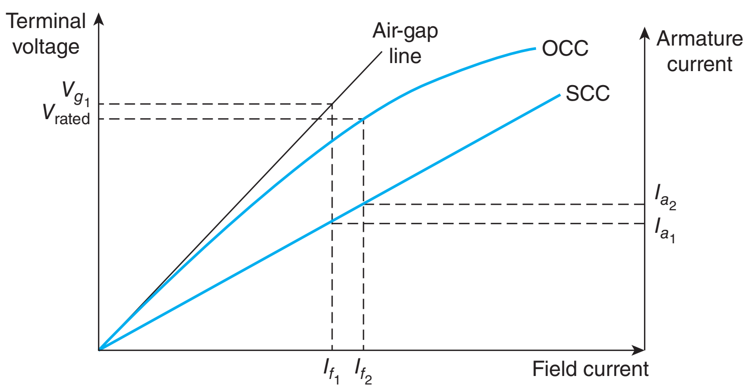


FIG. 5-26 Open- and short-circuit characteristics.

or

$$X_{s_{un}} = \left.\frac{V_{g_1}}{I_{a_1}}\right|_{I_{f1}} \tag{5.63}$$

Saturated Value

The ratio of the rated voltage (V_1) to the short-circuit current (I_{a_2}) at the same field conditions gives the approximate value of the saturated synchronous reactance ($X_{s_{sat}}$).

$$X_{s_{sat}} = \left.\frac{V_{rated}}{I_{a_2}}\right|_{I_{f2}} \tag{5.64}$$

The per-unit value of the saturated synchronous reactance is the reciprocal of the short-circuit ratio (SCR). This parameter is defined as follows:

$$\text{SCR} = \frac{\text{field current obtained from OCC at rated terminal voltage}}{\text{field current obtained from SCC at rated armature current}} \tag{5.65}$$

The Potier Triangle: Measurement of X_a and I'_a

The zero-power-factor (ZPF) characteristic is used in conjunction with the open-circuit characteristic in order to obtain the Potier triangle. The Potier triangle (Fig. 5-27) is used to determine:

The stator leakage reactance (X_a).

The armature reaction (I'_a) in equivalent field amperes.

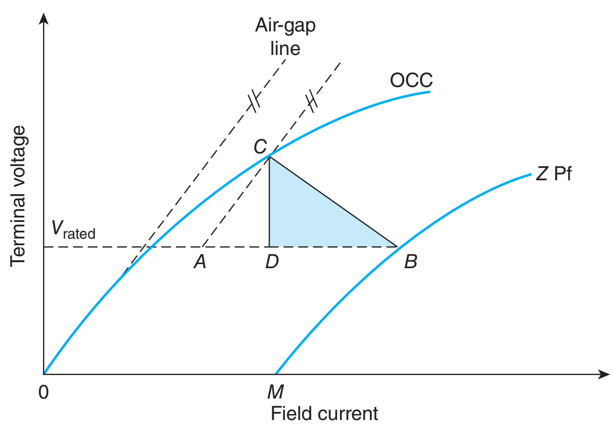


FIG. 5-27 Construction of the Potier triangle.

The following procedure is suggested in establishing the Potier triangle:

1. On the horizontal line at rated voltage, draw AB:

$$AB = OM \tag{5.66}$$

2. From point A, draw the line parallel to the air-gap line. This intersects the magnetization curve at point C. From this point, draw the perpendicular to AB. This establishes point D.

The base of this triangle gives the armature reaction in equivalent field amperes—that is, the I'_a. Thus,

$$BD = I'_a \tag{5.67}$$

The effective turns ratio (N_e) can be calculated from

$$N_e = \frac{I_a}{I'_a} \tag{5.68}$$

The other perpendicular side of the Potier triangle—that is, CD—gives the Potier reactance voltage drop. This is approximately equal to the voltage drop across the leakage reactance. Thus,

$$CD = I_a X_a \tag{5.69}$$

From this basis, the leakage reactance (X_a) can be calculated. Thus, the leakage reactance and the armature reaction are given, respectively, by the vertical and horizontal sides of the Potier triangle.

The origin of the ZPF characteristic corresponds to the value of field current required to circulate rated armature currents while the terminals of the machine are shorted. This particular point can also be obtained from the short-circuit characteristic.

Some Highlights of the Zero-Power-Factor Characteristic

Up to moderate saturation levels, the ZPF *lagging* characteristic can be obtained from the OCC by shifting the OCC downward by an amount equal to I_aX_a, and horizontally to the right by an amount equal to I'_a, as shown in Fig. 5-28(a). Similarly, the ZPF *leading* curve can be obtained from the OCC by shifting it horizontally to the left by an amount equal to I'_a, and vertically upward by an amount equal to I_aX_a, as shown in Fig. 5-28(a). The leakage reactance (X_a) is assumed to remain constant. At high saturation levels, this assumption is not valid.

When the power factor lags, more field current is required to magnetize the machine and to overcome the demagnetizing effects of the armature reaction.

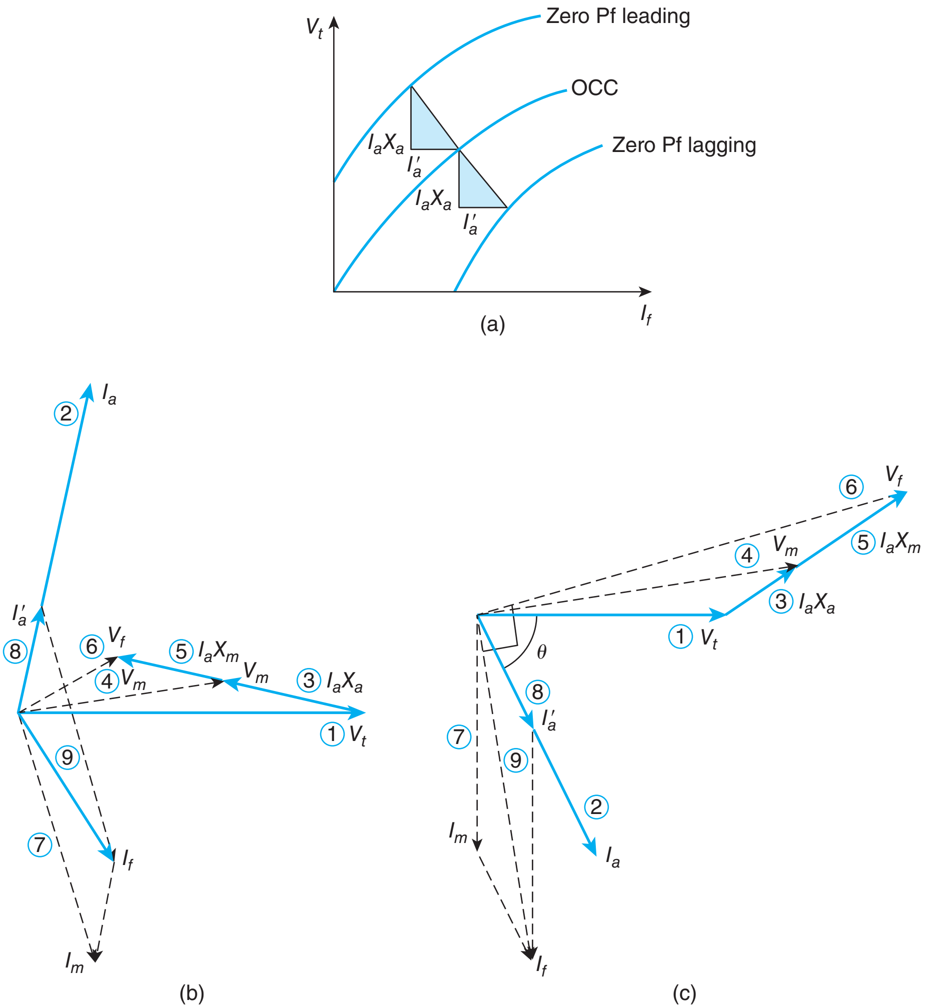


FIG. 5-28 Alternator: **(a)** zero-power-factor and open-circuit characteristics, **(b)** phasor diagram for near zero leading power factor, **(c)** phasor diagram for near zero lagging power factor.

When the power factor leads, less field current is required because the armature reaction aids the field in magnetizing the machine. At the origin of the zero-power-factor leading curve, the actual field current is zero because the armature reaction is equal and opposite to the magnetizing component of the field current.

The phasor diagrams for ZPF leading and lagging loads are shown in Figs. 5-28(b) and (c), respectively.

EXAMPLE 5-5

The three-phase alternator shown in the one-line diagram of Fig. 5-29(a) has a per-phase synchronous impedance of 0.03 + j0.50 per unit. The magnetization characteristic at the operating region is given by

$$V_f = 70 + 55I_f$$

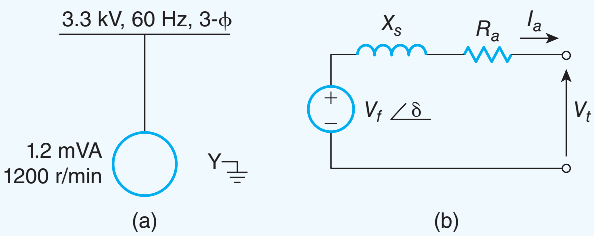


FIG. 5-29

The generator delivers rated current to an infinite bus at 0.8 leading power factor. Determine:

a. The ohmic value of the synchronous impedance.

b. The excitation voltage.

c. The approximate torque angle in electrical and mechanical degrees.

d. The regulation.

e. The field current.

SOLUTION

a. The base parameters are given by the name-plate data of the generator:

$$V_b = 5.3 \text{ kV}, \qquad S_b = 1.2 \text{ MVA}$$

Then, by definition,

$$Z_b = \frac{(3.3)^2}{1.2} = 9.08 \ \Omega/\text{phase}$$

The ohmic value of the synchronous impedance is calculated as follows:

$$Z_{\text{pu}} = \frac{Z_{\text{actual}}}{Z_b}$$

$$Z_{\text{actual}} = 9.08(0.03 + j0.50)$$

$$= 0.27 + j4.54 = \underline{\underline{4.55 \angle 86.6^\circ \ \Omega/\text{phase}}}$$

b. The magnitude of the rated current of the generator is

$$I = \frac{S}{\sqrt{3}\, V_{L\text{-}L}} = \frac{1200}{\sqrt{3}\, 3.3} = 209.95 \text{ A}$$

The excitation voltage is calculated by applying KVL in Fig. 5-29(b).

$$V_f \angle\delta = V_t + I_a Z_s$$

$$= \frac{3300}{\sqrt{3}} \angle 0^\circ + 209.95 \angle 36.9^\circ (4.55 \angle 86.6^\circ)$$

$$= 1592.8 \angle 30^\circ \text{ V/phase}$$

and

$$V_f = \underline{\underline{2758.8 \text{ V, line-to-line}}}$$

c. The approximate value of the torque angle in electrical degrees is

$$\delta = \underline{\underline{30^\circ}}$$

In mechanical degrees, its approximate value is

$$\delta_m = \frac{2\delta}{p} = \frac{2(30^\circ)}{6} = \underline{\underline{10^\circ}}$$

d. The regulation in percent is

$$R\% = \frac{2758.8 - 3300}{3300}(100) = \underline{\underline{-16.4\%}}$$

e. The field current is found from the given OC characteristic as follows:

$$I_f = \frac{2758.8 - 70}{55} = \underline{\underline{48.89 \text{ A}}}$$

EXAMPLE 5-6

Test results for a 3-ϕ, 60 Hz, 480 V, 400 kVA synchronous generator are as follows:

Open-circuit test:

Field current (A)	6	10	14	18	22	26
Armature voltage (V)	225	348	440	500	540	560

Short-circuit test: A field current of 8 A was required to produce the rated armature current.

Zero-power-factor test:

$$I_a = \text{rated}$$

$$V_{\text{L-L}} = 500\text{ V}$$

$$I_f = 30\text{ A}$$

Assuming that the stator resistance is negligible, calculate the field current required to produce the rated output voltage and current to a 0.90 Pf lagging load.

SOLUTION

The characteristics of the generator are shown in Fig. 5-30.

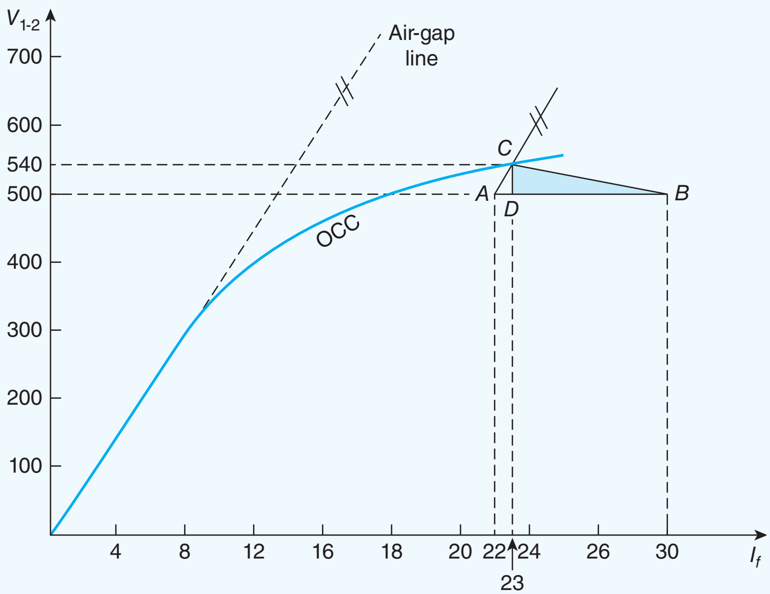


FIG. 5-30

From the diagram:

$$AB = 8\text{ A}$$

$$AC // \text{air-gap line}$$

From the Potier triangle:

$$CD = I_a X_a = 40 \text{ V, L-L}$$

$$DB = I_a' = 7 \text{ A}$$

The generator's rated current is

$$I_a = \frac{400}{\sqrt{3}(0.480)} = 481.13 \text{ A}$$

In phasor form,

$$I_a = 481.13\angle{-25.8°} \text{ A}$$

The leakage reactance is

$$X_a = \frac{40/\sqrt{3}}{481.13} = 0.048 \ \Omega/\text{phase}$$

The magnetization voltage is

$$V_m\angle\beta = \frac{480}{\sqrt{3}}\angle 0° + 481.13\angle{-25.8°}(j0.048)$$

from which

$$V_m = 287.95\angle 4.1° \text{ V/phase}$$

and

$$V_m = 498.74 \text{ V, L-L}$$

From the OCC, the magnetizing component of the field current is 17.95 A. From Eq. (5.51), we obtain

$$I_f = 17.95\angle{-85.9°} + 7\angle{-25.8°}$$

$$= \underline{\underline{22.3\angle{-70°} \text{ A}}}$$

Exercise 5-6

A 480 V, 50 kVA, 60 Hz, four-pole synchronous generator delivers rated power at 0.8 Pf lagging. The synchronous impedance is $0.2 + j1.4$ ohms per phase. Determine:

a. The excitation voltage.

b. The generator's regulation.

Answer (a) 593.38 V, L-L; (b) 23.62%

5.3 Salient-Pole Synchronous Machines

5.3.1 Introduction

A two-pole salient-rotor synchronous motor is shown in Fig. 5-31. The length of the air gap between the rotor and the stator is not uniform. The axis along which the air gap is minimum is called the *direct axis*, and that along which the air gap is maximum is called the *quadrature axis*.

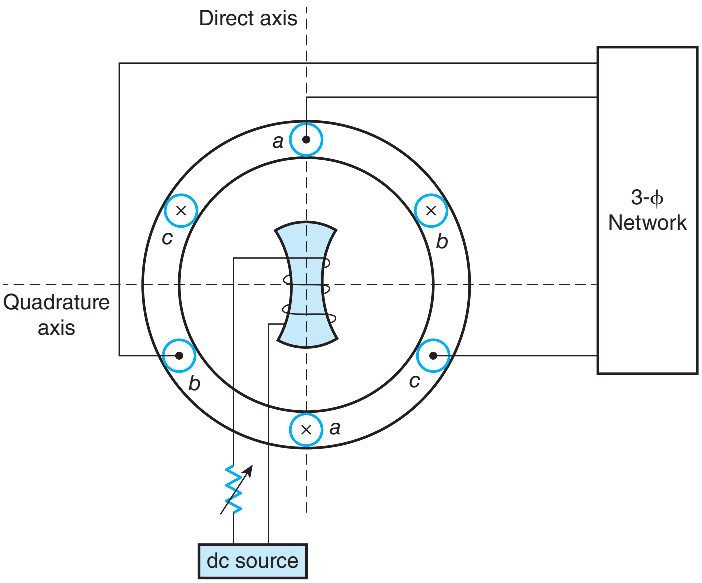


FIG. 5-31 An elementary representation of a two-pole salient-rotor synchronous machine.

Synchronous machines with salient-pole rotors usually have more than two poles and therefore rotate at relatively low synchronous speeds, as required by many industrial drives. They can develop up to 40% of their rated power without any excitation—zero field current—while cylindrical machines cease to function as motors or generators once the dc excitation is removed.

Saliency increases the ability of the machine to oppose the forces that tend to drive a motor out of synchronism. As a result, compared to cylindrical-rotor machines, salient machines are more suitable in applications where the motor might be subjected to occasional sudden torque variations.

In all modern synchronous motors, there is no direct connection between the rotor windings and the field supply voltage source. The field current, as explained in Chapter 7, is obtained by rectifying the voltage induced in another set of rotor windings, commonly known as a rotating armature.

The magnetizing flux through the machine's two distinct air gaps produces two different inductances. The inductance along the direct axis is designated as L_d, and that along the quadrature axis is designated as L_q. Since the inductance is an inverse function of the length of the air gap, the direct-axis (X_{m_d}) component of the magnetizing reactance is larger than its quadrature-axis (X_{m_q}) component. The ratio of these two reactances is about 1.5.

The effective reactance along either the direct (X_d) or the quadrature axis (X_q) is the sum of the armature leakage reactance (X_a) and its corresponding component of the magnetizing reactance. Mathematically,

$$X_d = X_{m_d} + X_a \tag{5.70}$$

$$X_q = X_{m_q} + X_a \tag{5.71}$$

where X_{m_d} and X_{m_q} are the direct- and quadrature-axis magnetizing reactances, respectively.

As mentioned in previous sections, the leakage reactance is very small compared to the components of the magnetizing reactance.

In the case of a cylindrical rotor:*

$$X_d = X_q = X_a \qquad \text{(synchronous reactance)} \tag{5.72}$$

To measure X_d and X_q, the excitation is removed from the machine and a voltage is applied across the stator terminals. Then the rotor is driven externally with a variable-speed dc motor. For different positions of the rotor magnetic axis relative to the stator axis, the maximum and minimum currents are recorded. The ratio of the applied voltage to the minimum and maximum currents gives, respectively, the direct-axis reactance and the quadrature-axis reactance. Under these test conditions, the machine is not saturated, and thus the measured reactances represent nonsaturated values.

Steady-State Analysis

The steady-state performance of salient-pole synchronous machines is satisfactorily predicted and analyzed by using Eq. (5.73). The positive signs are for a generator, and the negative signs are for a motor. This equation is derived from the De Blondel, or two-reaction, theory. This theory takes in the effects of saliency but ignores the effects of magnetic saturation. Magnetic saturation is accounted for by the cylindrical-rotor theory, which ignores saliency. Under normal operating conditions, both theories yield slightly different but generally satisfactory results.

*Cylindrical-rotor machines have some saliency, which exists because the slots for the field windings are not uniform around the entire rotor.

According to the De Blondel theory and as shown by Eq. (5.74), the armature current is represented by two components: the direct-axis component (I_d) and the quadrature-axis component (I_q). The relationship between the armature current and its components is not given in phasor form because the direct-axis component, depending on the power factor, may lead or lag the quadrature-axis component for either a motor or a generator. However, the two components are 90° out of phase with each other, and the quadrature-axis component is always in phase with the excitation voltage. All other parameters of Eq. (5.73) are the same as those for cylindrical-rotor synchronous machines.

$$V_f \underline{/\pm\delta^\circ} = V_t \underline{/0^\circ} \pm I_a R_a \pm jI_q X_q \pm jI_d X_d \tag{5.73}$$

$$|I_a| = \sqrt{I_q^2 + I_d^2} \tag{5.74}$$

The development of the per-phase equivalent circuit of a salient-pole machine from which Eq. (5.73) could be derived is rather complex, and it is not presented in this text.

5.3.2 Phasor Diagrams

The steady-state analysis of salient-pole synchronous machines is simplified by using phasor diagrams—that is, the graphical representation of Eqs. (5.73) and (5.74). The following procedure is suggested in drawing the phasor diagrams for a motor.

From Eq. (5.73), write the governing voltage equation:

$$V_f \underline{/-\delta^\circ} = V_t \underline{/0^\circ} - I_a R_a - jI_q X_q - jI_d X_d \tag{5.75}$$

1. Draw to scale the terminal voltage (V_t) and take it as a reference (see Fig. 5-32).
2. Draw the armature current at its given phase angle. In this case, $\theta = 0^\circ$.
3. From the tip of the terminal voltage, draw the phasor $I_a R_a$. This is the voltage drop in the resistance (R_a) and should be in phase with the armature current (I_a). Here, however, $I_a R_a$ must be drawn at 180° to its normal direction because of the negative sign in the voltage equation.
4. Draw to scale a dotted line representing the phasor $I_a X_q$. This is the voltage drop in the reactance (X_q) and should lead the current (I_a) by 90°. Here again $I_a X_q$ must be drawn at 180° to its normal direction because of the minus sign in the voltage equation. The end of this phasor establishes the excitation line (V_f line) and therefore the torque angle. Although it is not in the equation, the term $I_a X_q$ is used to locate the excitation or quadrature-axis line. For this reason, it is drawn with a dotted line. The torque angle can also be obtained mathematically from the following relationship:

$$V_q \underline{/-\delta^\circ} = V_t \underline{/0^\circ} - I_a R_a - jI_a X_q \tag{5.76}$$

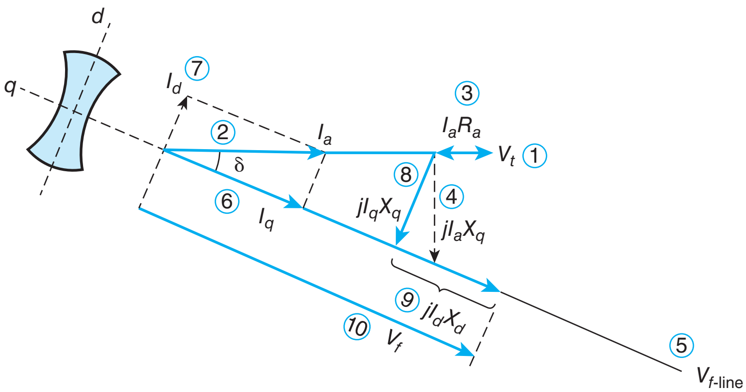


FIG. 5-32 Phasor diagram for a salient synchronous motor operating at unity power factor.

where V_q represents the sum of the three phasors of Eq. (5.76). Its phase angle (δ) establishes the direction of the line on which the excitation voltage must lie.

5. Draw the quadrature or excitation line. This is designated as the V_f line.
6. Resolve the armature current into its rectangular components I_d and I_q. The quadrature-axis component (I_q) is always in phase with the excitation line.
7. The direct-axis current is 90° out of phase with the quadrature axis. In general, I_d may lead or lag I_q, depending on the machine's operating power factor.
8. Draw the phasor I_qX_q. This represents the voltage drop across the reactance (X_q) and should lead the current by 90°. Again, because of the negative sign in the voltage equation, it must be drawn at 180° to its normal direction. The length of I_qX_q does not need to be calculated because it always starts at the tip of I_aR_a and ends perpendicularly on the excitation line.
9. Finally, draw to scale the phasor I_dX_d. As this represents the voltage drop in the reactance (X_d), it should lead the current (I_d) by 90°. But because of the equation's minus sign, it is drawn at 180° to its normal direction.
10. The excitation voltage (V_f) is drawn from the origin to the end of the tip of the phasor I_dX_d.

The procedure for drawing the phasor diagrams for salient alternators is similar. In fact, it is simpler to construct the phasor diagrams for a generator because of the positive sign in the loop Eq. (5.73).

The phasor diagram of a salient synchronous generator operating at unity power factor is shown in Fig. 5-33.

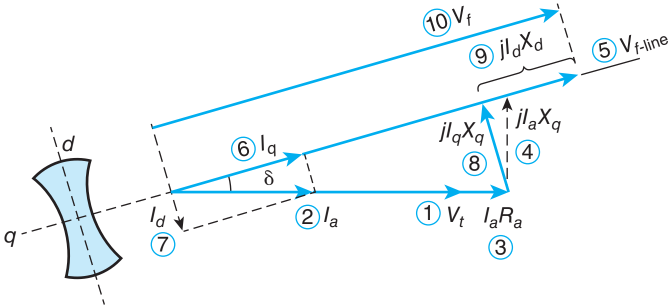


FIG. 5-33 Phasor diagram for salient synchronous generator operating at unity power factor.

5.3.3 Power Developed

To find the expression for the power developed by a salient synchronous motor, consider the phasor diagram shown in Fig. 5-34. Neglect armature resistance and assume a leading power factor. By definition,

$$P = V_t I_a \cos\theta \text{ watts/phase} \tag{5.77}$$

Say,

$$\beta = \theta + \delta \tag{5.78}$$

Then

$$P = V_t I_a \cos(\beta - \delta) \tag{5.79}$$

$$= V_t I_a \cos\beta \cos\delta + V_t I_a \sin\beta \sin\delta \tag{5.80}$$

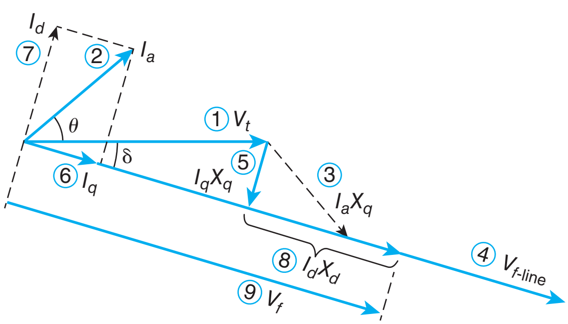


FIG. 5-34 Phasor diagram for a leading power factor synchronous motor.

From the phasor diagram,

$$I_q = I_a \cos \beta \tag{5.81}$$

$$I_d = I_a \sin \beta \tag{5.82}$$

Substituting, we get

$$P = V_t I_q \cos \delta + V_t I_d \sin \delta \tag{5.83}$$

From the phasor diagram, it is also evident that

$$I_d = \frac{V_f - V_t \cos \delta}{X_d} \tag{5.84}$$

and

$$I_q = \frac{V_t \sin \delta}{X_q} \tag{5.85}$$

Substituting Eqs. (5.85) and (5.84) into Eq. (5.83), we obtain

$$P = V_t \frac{V_t \sin \delta}{X_q} \cos \delta + \frac{V_t \sin \delta (V_f - V_t \cos \delta)}{X_d} \tag{5.86}$$

From the above,

$$P = \frac{V_t^2 (X_d - X_q)}{X_d X_q} \sin \delta \cos \delta + \frac{V_f V_t}{X_d} \sin \delta \tag{5.87}$$

using

$$\sin 2\delta = 2 \sin \delta \cos \delta \tag{5.88}$$

The expression in Eq. (5.87) becomes

$$P = \frac{V_f V_t}{X_d} \sin \delta + \frac{V_t^2 (X_d - X_q)}{2 X_d X_q} \sin 2\delta \text{ watts/phase} \tag{5.89}$$

The first term of Eq. (5.89) is identical to Eq. (5.28), which represents the power developed by a cylindrical synchronous machine. The second term represents the saliency and is referred to as the reluctance component. This component may account for up to 40% of a salient machine's output power. The power developed versus the torque angle is shown in Fig. 5-35.

By definition, the reactive power is

$$Q = V_t I_a \sin \theta \tag{5.90}$$

As before,

$$\theta = \beta - \delta \tag{5.91}$$

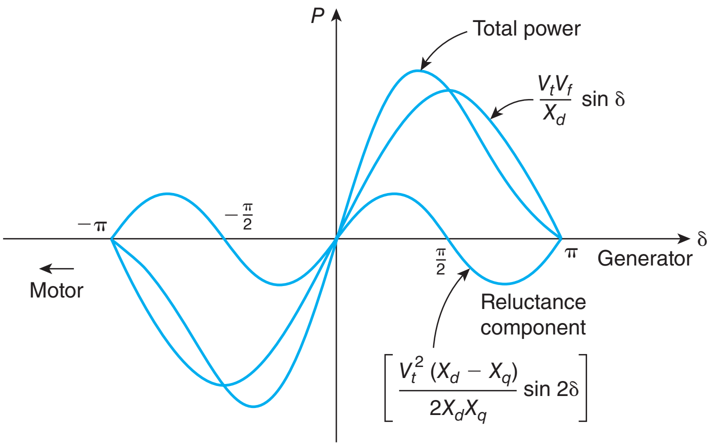


FIG. 5-35 Power versus torque angle.

Thus,

$$Q = V_t I_a \sin(\beta - \delta) \tag{5.92}$$

From Eqs. (5.84), (5.85), and (5.92), after expansion and simplification, we get

$$Q = \frac{V_t V_f}{X_d}\cos\delta - V_t^2\left(\frac{\cos^2\delta}{X_d} + \frac{\sin^2\delta}{X_q}\right) \text{VAR/phase} \tag{5.93}$$

When the parameters of the machine are expressed in per unit, Eqs. (5.89) and (5.93) give the total input real and reactive power in perunit.

5.3.4 Torque Angle for Maximum Power

When the motor is supplied from an infinite bus and with a constant field current, then Eq. (5.89) can be written as

$$P = K_1 \sin\delta + K_2 \sin 2\delta \tag{5.94}$$

where the constants K_1 and K_2 are given, respectively, by

$$K_1 = \frac{V_f V_t}{X_d} \tag{5.95}$$

$$K_2 = \frac{V_t^2 (X_d - X_q)}{2X_d X_q} \tag{5.96}$$

In order to find the torque angle that corresponds to the maximum power, apply the calculus theory of maximum and minimum, as outlined below:

1. Find the derivative of the power with respect to the torque angle:

$$\frac{dP}{\partial\delta}$$

2. Set this derivative equal to zero:

$$\frac{dP}{d\delta} = 0$$

3. From the last expression, find the so-called critical values of δ.

Applying this procedure, and after some mathematical manipulations, we find that the torque angle for maximum power is given by the following equation:

$$\delta = \arccos\left(-\frac{K_1}{8K_2} \pm \sqrt{\left(\frac{K_1}{8K_2}\right)^2 + \frac{1}{2}}\right) \tag{5.97}$$

The torque angle for maximum power is usually about 70°. The intermediate steps in the derivation of Eqs. (5.93) and (5.97) are left as an exercise (see Exercise 5-7(b)).

5.3.5 Stiffness of Synchronous Machines

The stiffness or toughness of a synchronous machine is a measure of its ability to resist and oppose the forces that tend to pull it out of synchronism. Mathematically, the stiffness is given by the partial derivative of the power developed with respect to the torque angle. In other words, the stiffness of a machine is given by the slope of the power-angle curve at the operating point under consideration. Maximum stiffness is referred to as synchronizing power.

Cylindrical Rotors

From Eq. (5.28), we can easily obtain the stiffness (S_T) of a cylindrical-rotor machine as follows:

$$S_T = \frac{dP}{d\delta} = 3\frac{V_f V_t}{X_s}\cos\delta \text{ watts/elect. radian} \tag{5.98}$$

where the factor of 3 accounts for the three phases of the machine.

The stiffness is maximum at the origin of the power-angle curve and minimum at the torque angle that corresponds to zero output power. In other words, at pull-out, the stiffness of the machine is zero.

From Eq. (5.89), it can be shown that the stiffness of a motor with salient rotor is

$$S_T = \frac{dP}{d\delta} = 3\left(\frac{V_f V_t}{X_d}\cos\delta + \frac{V_t^2(X_d - X_q)}{X_d X_q}\cos 2\delta\right) \text{watts/elect. radian} \tag{5.99}$$

Although an increase of the field current does not affect the machine's output power, it does increase its toughness.

EXAMPLE 5-7

The synchronous motor shown in the one-line diagram of Fig. 5-36(a) draws rated current at a power factor of 0.95 leading. Its direct- and quadrature-axis reactances are 0.8 per unit and 0.5 per unit, respectively. Neglecting losses, determine:

1. a. The approximate torque angle in electrical and mechanical degrees.
   b. The excitation voltage.
   c. The maximum torque that the machine can develop when the excitation voltage, as determined in (b), is constant.
2. Repeat part 1 by using the cylindrical rotor theory.

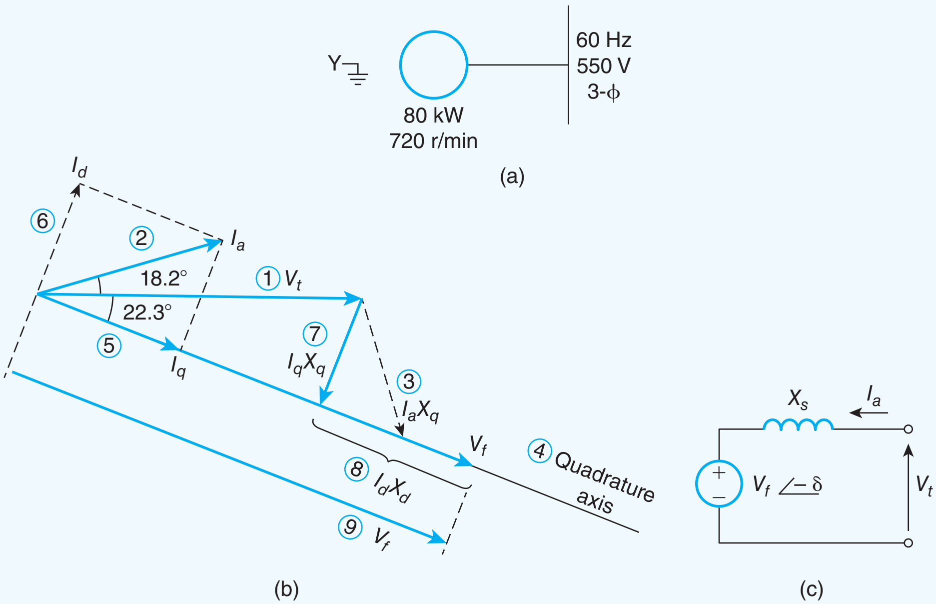


FIG. 5-36

SOLUTION

1. a. The torque angle is found by using Eq. (5.76).

$$V_q \angle -\delta = V_t - jI_aX_q$$

$$V_q \angle -\delta = 1.0 \angle 0° - j1.0 \angle 18.2° (0.5) = 1.25 \angle -22.3° \text{ pu}$$

Thus, the torque angle in electrical degrees is

$$\delta = \underline{\underline{-22.3°}}$$

In mechanical degrees, it is

$$\delta_m = \frac{2}{10}(22.3) = \underline{\underline{4.5°}}$$

b. The phasor diagram, as shown in Fig. 5-36(b), is drawn by following the procedure outlined in Section 5.3.2. Then, from basic trigonometry,

$$I_q = I_a \cos(\theta + \delta) \angle -\delta$$

$$= \cos(18.2° + 22.3°) \angle -22.3° = 0.76 \angle -22.3° \text{ pu}$$

and

$$I_d = I_a \sin(18.2° + 22.3°) \angle 90° - 22.3°$$

$$= 0.65 \angle 67.7° \text{ pu}$$

Using Eq. (5.73), we obtain

$$V_f \angle -\delta = 1.0 \angle 0° - j0.76 \angle -22.3° (0.5) - j0.65 \angle 67.7° (0.8)$$

$$= \underline{\underline{1.45 \angle -22.3° \text{ pu}}}$$

c. The torque angle for maximum power is found from Eq. (5.97):

$$K_1 = \frac{1.45(1)}{0.8} = 1.81$$

$$K_2 = \frac{(1.0)^2 (0.8 - 0.5)}{2(0.8)(0.5)} = 0.375$$

$$\delta = \arccos\left(-\frac{1.81}{8(0.375)} \pm \sqrt{\left(\frac{1.81}{8(0.375)}\right)^2 + \frac{1}{2}}\right)$$

from which

$$\delta = 70.9°$$

The maximum torque is

$$T = \frac{\text{maximum power in perunit}}{\text{speed perunit}}$$

$$T = \frac{1.45(1)}{0.8}\sin 70.9^\circ + \frac{(1.0)^2(0.8 - 0.5)}{2(0.8)(0.5)}\sin 141.9^\circ$$

$$= \underline{\underline{1.94 \text{ pu}}}$$

2. For a cylindrical rotor,

$$X_d = X_q = X_s = 0.8 \text{ pu}$$

a. Using the per-phase equivalent circuit of Fig. 5-36(c), from KVL, we have

$$V_f \angle -\delta = 1.0 \angle 0^\circ - j1.0 \angle +18.2^\circ (0.8) = 1.46 \angle -31.3^\circ \text{ pu}$$

Thus,

$$\delta = \underline{\underline{-31.3^\circ}}$$

and

$$\delta_m = \frac{2}{10}(31.3) = \underline{\underline{6.3^\circ}}$$

b. The excitation voltage is

$$V_f = \underline{\underline{1.46 \text{ pu}}}$$

c. For maximum torque,

$$\delta = 90^\circ$$

and

$$T = \left(\frac{1}{1.0}\right)\frac{1.46(1)}{0.8} = \underline{\underline{1.83 \text{ pu}}}$$

For purposes of comparison, the results are summarized in Table 5-3.

TABLE 5-3 Summary of results of Example 5-7

Type of rotor	*Torque Angle in Electrical Degrees*	*Excitation Voltage in pu*	*Maximum Torque in pu*
Salient	22.3	1.45	1.94
Cylindrical	31.3	1.46	1.83

Exercise 5-7

a. Draw the phasor diagram of a salient synchronous motor operating at unity power factor and show that the power angle is given by

$$\delta = \arctan \frac{I_a X_q}{V_t - I_a R_a}$$

b. Derive Eqs. (5.93) and (5.97).

c. Determine the torque angle for maximum power of a 480 V, 50 kW, 0.9 efficient, unity power factor, salient synchronous motor. The motor's direct- and quadrature-axis reactances are 3 Ω and 2 Ω, respectively.

Answer (c) 71°.

5.4 Conclusion

The operation of synchronous motors is based on the natural tendency of two fields to try to align their magnetic axes. The two fields—the stator and rotor fields—rotate at synchronous speeds.

The rotating stator field is produced by balanced three-phase currents flowing in properly distributed stator coils. The rotor field is produced by the direct current that flows through the synchronously rotating rotor windings.

The speed of a synchronous motor is directly proportional to the frequency of the stator voltage and inversely proportional to the number of poles. For a fixed-frequency source and number of poles, the speed is also fixed.

Because ordinary synchronous motors have these two crucial limitations—they operate at only one speed, and they require a dc voltage for their rotor windings and a balanced three-phase voltage for their stator windings—they have few applications.

At starting, synchronous motors function like squirrel-cage induction machines and thus draw large inrush currents. As a result, the voltage drop in the impedance of the supply network may be considerable. The higher the inrush current, the greater the disturbance on the other electrical apparatus operating within the same distribution network.

High-inrush currents also *reduce* the starting torque to levels that are sometimes too low to meet the requirements of a connected load. In this case, an engineer may select a soft start motor. Such motors have relatively low starting torque and current, and they produce sufficient power to provide the high starting-torque requirements of some loads.

The torque produced by a synchronous motor depends on its terminal voltage, its excitation voltage, and the phase angle between these two voltage phasors (commonly known as the power angle). Under normal operating conditions, the power angle is usually less than 30°.

The stator field may aid or oppose the rotor field. Once a machine is designed, this armature reaction depends on the operating power factor and on the stator current. As a result, a leading-power-factor motor requires more field current than a lagging-power-factor motor. Conversely, in order to produce a given terminal voltage, a synchronous generator requires much more field current when it operates at a lagging power factor than when it operates at a leading power factor. In either case, it is always useful to sketch and inspect a phasor diagram before arriving at any conclusions. As seen from the ac voltage supply, synchronous machines appear as variable capacitive or inductive impedances.

Vee curves give the variation of the power factor, and the armature current drawn by the motor, as a function of field current. In general, synchronous motors should not be designed for, or be operated at, leading power factors because their stator copper losses are thereby increased. It is much more economical to operate at unity power factor and to purchase a capacitor bank to provide the required lagging kVAR power. Reducing the power factor of a synchronous motor has a disadvantage, however: Increasing the power factor from 0.8 leading to unity decreases the breakdown torque of the motor. Depending on load requirements, the motor may therefore stall. In short, each particular situation should be analyzed before any arbitrary adjustments are made.

Synchronous generators are used exclusively to generate three-phase power. Utility companies use large units (up to several hundred MVAs) to generate commercially available three-phase power. Units with lower ratings (up to 1 MVA) are used in various plants to generate the standby power required to operate critical loads when the normal power source is temporarily interrupted. These generators are always driven at constant speeds by so-called prime movers.

Theoretically, depending on the field current and power factor, the terminal voltage of a generator varies over a wide range. In practice, however, owing to closed-loop voltage-regulator systems, this voltage is kept constant at the desired level.

The various circuit parameters of a synchronous machine, when expressed in per unit (as is the practice), are within a closed range and are available through a manufacturer's published data.

Salient-pole machines are stiffer than cylindrical-rotor machines. Being able to tolerate stronger load disturbances, they normally operate at speeds below 1800 r/min. Salient machines can be analyzed using the graphical or mathematical interpretation of the De Blondel equation. Salient machines can develop up to 40% of their rated torque without any excitation, whereas cylindrical-rotor machines cease to operate when the field current is removed.

Synchronous motors are not used (except when they are equipped with flywheels) to drive loads whose torque changes abruptly or is cyclical in nature because they can become unstable. Their stiffness, or capacity to oppose external disturbances, depends on their degree of saliency. Damping windings provide a higher starting torque and a stabilizing force each time the motor is forced to deviate from its synchronous speed.

Fig. 5-37 shows the motor torque and load characteristic versus speed for a 2800 kW synchronous motor.

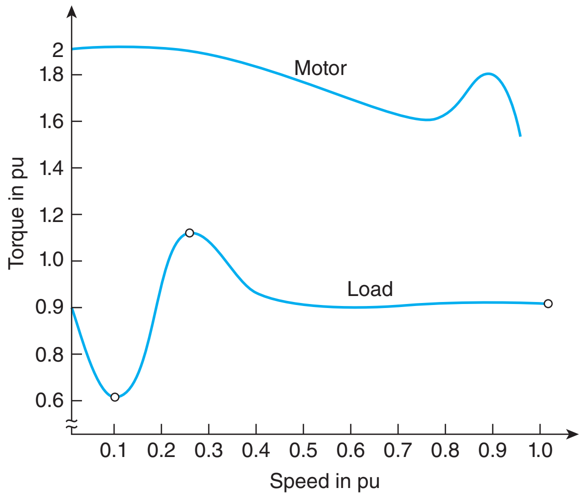


FIG. 5-37 Load and motor torque-speed characteristics. Motor: 2800 kW, 225 r/min, 60 Hz, 4000 V synchronous motor. Load: Ball grinding mill.

5.5 Review of Important Mathematical Relationships

Table 5-5 summarizes the main concepts of this chapter in their mathematical form.

TABLE 5-5 Review of important mathematical relationships

Item	*Description and Formula*	*Remarks*
	a. General	
1	Strength of the synchronously rotating field	
	$\mathscr{F}_s = \frac{3}{2}\mathscr{F}_1 \cos(\omega t - \beta)$	Eq. (5.1)
2	Speed of synchronous machines	
	$n_s = 120\frac{f}{p}$ r/min	Eq. (5.2)
	b. Cylindrical Rotor	
3	Synchronous impedance	
	$Z_s = R_a + jX_s$	Eq. (5.7)

TABLE 5-5 *(Continued)*

Item	*Description and Formula*	*Remarks*
4	Relationship between mechanical and electrical radians (power angle)	
	$\delta_m = \frac{2}{p}\delta$	Eq. (5.8)
5	Effective turns ratio	
	$N_e = \frac{I_a}{I'_a}$	Eq. (5.9)
6	Field current	
	$I_f = I_m - I'_a$	Eq. (5.11)
7	Excitation voltage	
	$V_f \angle -\delta = V_t \angle 0° - I_a (R_a + jX_s)$	Eq. (5.12)
8	Power developed ($R_a = 0$)	
	$P = \frac{V_t V_f}{X} \sin\delta$ watts/phase	Eq. (5.28)
9	Reactive power ($R_a = 0$)	
	$Q = \frac{V_t}{X}(V_t - V_f \cos\delta)$	Eq. (5.36)
	c. Three-phase Generators	
10	Excitation voltage	
	$V_f \angle \delta° = V_t \angle 0° + I_a(R_a + jX_s)$	Eq. (5.50)
11	Field current	
	$I_f = I_m + I'_a$	Eq. (5.51)
12	Effective turns ratio	
	$N_e =$ slope of SCC	Eq. (5.58)
	$= \frac{I_a}{I'_a}$	
13	Saturated value of synchronous reactance	Eq. (5.64)
	$X_{s_{sat}} = \frac{V_{rated}}{I_{a_2}}\Big\vert_{I_{f2}}$	

(*Continues*)

TABLE 5-5 *(Continued)*

Item	Description	Remarks
	d. Salient-Pole Machines	
14	Excitation voltage	
	$V_f \angle \pm\delta° = V_t \angle 0° \pm I_a R_a \pm jI_q X_q \pm jI_d X_d$	Eq. (5.73)
	(positive sign for generator; negative sign for motor)	
15	Power developed	
	$P = \dfrac{V_f V_t}{X_d} \sin\delta + \dfrac{V_t^2 (X_d - X_q) \sin 2\delta}{2X_d X_q}$ watts/phase	Eq. (5.89)

5.6 Manufacturer's Data

Tables 5-6, 5-7, and 5-8 present typical machine data, as furnished by manufacturers.

TABLE 5-6 Estimated characteristics for a three-phase, 514 r/min, 60 Hz, 4000 V, 3300 kW salient-pole synchronous motor—soft start—brushless exciter

		Power Factor Leading	
Item	Parameter	0.90	0.80
1	Inrush current in percent	370	340
2	Torque in percent:		
	Starting	50	50
	Pull-in	65	62
	Pull-out	175	195
3	Efficiency in percent:		
	Full-load	95.2	94.8
	$\frac{1}{2}$ full-load	94.2	93.8
4	Reactances in per unit:		
	X_d	2.2	2.46
	X'_d	0.45	0.55
	X''_d	0.28	0.32
5	Field excitation in kW	40	40

Based on data from General Electric Canada, Inc.

TABLE 5-7 Data for a three-phase, 225 r/min, 4000 V, 0.80 power factor leading. 2800 kW synchronous motor—hard start—brushless exciter

Item	Parameter	Value	Remarks
1	Inrush current in percent	600	The starting characteristics of the motor and load (ball grinding mill) are shown in Fig. 5-37.
2	Torque in per unit:		
	Starting	2.2	
	Pull-in	1.4	
	Maximum	275	
3	Reactances in per unit:		
	X_d	1.1	
	X'_d	0.29	
	X''_d (unsaturated value)	0.126	
4	Efficiency at full-load	0.95	
5	Exciter's amps at rated load	13.9	
6	Field resistance at 25°C in ohms	6.32	
7	Allowable stall time in seconds	5	
8	Accelerating time in seconds	4.5	

Based on data from General Electric Canada, Inc.

TABLE 5-8 Data for high- and low-starting-torque synchronous motors: 514 r/min, 60 Hz, 0.8 leading Pf, 4500 kW

		Type of Starting Torque	
Item	Description	High	Low
1	Per-unit cost[†]	1.0	1.15
2	Inrush current in per unit	6.5	3.75
3	Power factor at starting in percent	28	12
4	Efficiency at full-load in percent	96	96.5
5	Torques in per unit:		
	Starting	1.8	0.4
	Pull-in	1.4	0.7
	Pull-out	2	2
6	Torque angle at nominal operating conditions	28°	27°
7	Relative diameters	1	1.2

[†]*In 2010, the per-unit cost was about $600,000.*

Based on data from General Electric Canada, Inc.

5.7 Review Questions

1. What are the advantages and disadvantages of synchronous motors?
2. Explain the magnetizing and demagnetizing effects of the armature current.
3. Draw and explain the Vee curves for a synchronous machine.
4. When starting a synchronous motor, a small resistance is placed in the rotor circuit; otherwise, the open-circuited windings may be damaged. Explain the reasons for this.
5. What conditions are necessary to produce a synchronously rotating magnetic field? How does this field compare with that produced by a permanent magnet rotating at the same speed?
6. The excitation voltage in a synchronous machine can be either larger or smaller than the terminal voltage, depending on the power factor. What are the reasons for this?
7. What is torque angle, and on what external factors does it depend? How could it be observed in a laboratory?
8. What does the Potier triangle give? How would you measure it?
9. Define voltage regulation, and explain its significance and effects on the operation of electrical equipment.
10. What are the advantages and limitations of synchronous machines with salient rotors?
11. How does the impedance of a transmission line affect the power that can be transmitted between a generator and an infinite bus?

5.8 Problems

5-1 A three-phase, 60 Hz, 10-pole, 480 V, 100 kW synchronous motor has an overall efficiency of 0.92 and draws rated current at unity power factor. Its per-phase synchronous impedance is $0.10 + j1.1$ ohms. Determine:

- **a.** The excitation voltage.
- **b.** The approximate value of the torque angle, in mechanical degrees.
- **c.** The stator electrical losses and the rotational losses.

5-2 A 300 kW, 4.16 kV, 600 r/min, 60 Hz, 3-ϕ synchronous motor has a synchronous impedance of $1 + j5$ ohms/phase. The motor delivers rated power and draws nominal current at a power factor of 0.9 leading. Its rotational losses are 12 kW. Determine:

- **a.** The armature current.
- **b.** The excitation voltage and the torque angle.
- **c.** The per-unit value of the synchronous impedance.

5-3 A 230 V, three-phase, 60 Hz, eight-pole, 37.3 kW star-connected synchronous motor has a synchronous reactance of 0.60 ohms per phase. The rotational losses are constant at 1800 watts, and the stator resistance is negligible. Data for the open-circuit characteristic, at rated speed, is as follows:

Line-to-line voltage (V)	138	228	292	332	347
Field current (A)	2.0	4	6	8	10

Determine:

a. The field current when the motor draws rated current at a power factor of 0.80 leading.

b. The armature current when the load is halved and the excitation remains the same as in (a).

5-4 A 300 kW, four-pole, 2.3 kV, 60 Hz, 3-ϕ synchronous motor has a synchronous impedance of $0.3 + j4.5$ ohms/phase. Determine:

a. The torque angle at which the motor will draw maximum power from the infinite bus.

b. The torque angle at which the motor will draw maximum reactive power from the infinite bus.

5-5 A 750 kW, 1200 r/min, 2.2 kV, 60 Hz, 3-ϕ, 0.965 efficient synchronous motor has a per-phase synchronous reactance of 3.0 ohms and delivers power to a constant-torque mechanical load. The open-circuit characteristic at the operating region is given by

$$V_f = 254.37I_f$$

When the motor draws rated current at unity power factor, determine:

a. **1.** The excitation voltage.

2. The torque angle.

3. The field current.

4. The magnitude of the armature current.

5. The real, reactive, and complex power drawn by the motor.

b. If the excitation is changed by ±25% from its nominal value, while the load remains constant, repeat (a).

5-6 A 300 kW, 2-pole, 1.2kV, 60 Hz, 3-ϕ synchronous motor has a leakage impedance of $0.1 + j1.0$ ohms/phase. The effective turns ratio per pole is 20:1. The open-circuit characteristic is given by

$$V_f = 30 + 55I_f$$

The motor receives electrical power at unity power factor. Its rotational losses are 17.05 kW. Determine:

a. The magnetization voltage.

b. The magnetizing component of the field current.

c. The armature reaction in equivalent field amperes.

d. The field current and the exact torque angle, in mechanical degrees.

e. The voltage induced in the armature windings due to the field current.

5-7 A 500 kVA, 1800 r/min, 4.16 kV, 60 Hz, 3-ϕ, 0.94 efficient synchronous machine supplies rated current at 0.80 power factor lagging. Its synchronous impedance per phase is $1 + j12$ ohms. Determine:

a. The number of poles.

b. The per-unit value of the synchronous impedance.

c. The stator copper loss.

d. The electromechanical torque developed.

5-8 **a.** A 1.2 MVA, 1800 r/min, 6.6 kV, 60 Hz, 3-ϕ generator delivers rated current to an infinite bus at unity power factor. The synchronous reactance is 1.0 per unit, based on its own rating. The open-circuit characteristic in the operating region is given by $V_f = 100 + 65I_f$. Determine:

1. The excitation voltage.

2. The torque angle.

3. The armature current.

4. The field current.

5. The complex power delivered to the load.

b. Repeat part (a) if the field current is kept constant while the power supplied by the prime mover is changed by ±25%.

5-9 A 750 kW, 1800 r/min, 6.3 kV, 60 Hz, 3-ϕ, 0.94 efficient synchronous motor when tested gave the following results:

Open-circuit test at rated speed:

Line voltage in volts	3500	5500	6500	7500	8000	8500
Field current in amperes	6	9.6	11.8	15	17.2	22

Short-circuit test at rated speed:

Line current	105 A
Field current	9 A

Zero-power-factor test:

Line current	105 A
Field current	20 A
Line voltage	5900 V

The motor delivers rated power and operates at a power factor of 0.80 leading. The resistance of the armature is 1.35 Ω/phase. Determine:

a. The Potier triangle.

b. The effective turns ratio.

c. The field current.

5-10 A 373 kW, 480 V, 0.8 Pf leading salient-pole synchronous motor has a direct-axis synchronous reactance of 1.0 per unit and a quadrature-axis synchronous reactance of 0.65 per unit. The rotational and stator copper losses are negligible. The field current is adjusted so that the motor draws minimum armature current while delivering rated power to a mechanical load. Under this operating condition, determine:

a. The power factor.

b. The power angle.

c. The excitation voltage.

d. The power developed. What percent of the total power is reluctance power?

5-11 A 5 MVA, 600 r/min, 6.6 kV, 60 Hz, 3-ϕ synchronous alternator delivers rated current at a power factor of 0.95 leading. Its direct-axis reactance and quadrature-axis reactance are 8.0 and 6.0 ohms, respectively. Determine:

a. The torque angle at full-load.

b. The regulation.

c. The reluctance torque in percent of the rated torque.

5.12 Explain the following:*

a. Increasing the load to a power generating system, the frequency of the distribution system is momentarily decreased, while on decreasing the load, the frequency is increased.

(This frequency variation is used in the momentary control of the power output in a multi-machine generating station when the power of any of the supplied loads is suddenly changed).

b. Generating systems that include rotating inertia are less sensitive to electrical disturbances than those which do not include it (batteries, photo voltaics, etc.).

c. Utilities without downstream reactive power cannot push real power towards consumers.

* Extensive articles on wind power generation and photovoltaics one may find in "Transmission and Distribution World", July 2013.